25

THE ELECTRIC POTENTIAL

Conceptual Questions

25.1. **(a)** Use the expression for the potential energy of a point charge.

$$\frac{U_1}{U_2} = \frac{\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 Q}{r}}{\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_2 Q}{2r}} = \frac{q_1(2r)}{q_2(r)} = \frac{q_1(2r)}{\left(\dfrac{q_1}{3}\right)(r)} = 6$$

(b) Use the expression of the potential energy of a point charge in a parallel plate capacitor.

$$\frac{U_1}{U_2} = \frac{q_1 Es}{q_2 E(2s)} = \frac{q_1 s}{\left(\dfrac{q_1}{3}\right)2s} = \frac{3}{2}$$

25.2. **(a)** The electric force is conservative. Since $\Delta K = -\Delta U$, $K_\mathrm{f} - K_\mathrm{i} = U_\mathrm{i} - U_\mathrm{f}$. We have $K_\mathrm{f} = 0$ and $U_\mathrm{i} = 0$. Thus $K_\mathrm{i} = U_\mathrm{f}$. Reading from the graph, at 10 fm $U_\mathrm{f} = 2\times10^{-12}$ J.

(b) At 20 fm, K + U = total energy $= 2\times10^{-12}$ J. Reading from the graph, U(20 fm) $= 1\times10^{-12}$ J. Thus K + $(1\times10^{-12}$ J) = 2×10^{-12} J, so K $= 1\times10^{-12}$ J.

25.3. **(a)** The electric potential energy of the charges is $U = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r}$. The potential energy varies as $\dfrac{1}{r}$, but because the two charges have opposite signs the potential energy actually increases as r increases.
(b) Less than. Two opposite charges slow down as they move farther apart.

25.4. **(a)** $\Delta U_{1\to2} = \Delta U_{1\to3}$. $\Delta U = qEs$ where s is the distance measured vertically from point 1 to point 2. Only the vertical displacement matters here because there is no horizontal component of E or of the force.
(b) $v_2 = v_3$. The change in kinetic energy $\Delta K_{1\to2}$ is equal to $\Delta K_{1\to3}$ because $\Delta U_{1\to2} = \Delta U_{1\to3}$ and the total energy is constant.

25.5. $U_a > U_c > U_f = U_e > U_b > U_d$. Calculate: $U = -\vec{p}\cdot\vec{E} = -pE\cos\phi$.

$U_a = -pE\cos180° = +pE$	$U_d = -pE\cos0° = -pE$
$U_b = -pE\cos45° = -0.707pE$	$U_e = -pE\cos90° = 0$
$U_c = -pE\cos135° = +0.707pE.$	$U_f = -pE\cos(-90°) = 0.$

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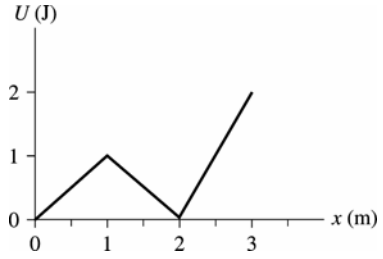
25.6. **(a)** Note $U = qV$.



(b) At $x = 1$ m, the total energy $E = K_1 + U_1 = 1\text{ J} + 1\text{ J} = 2\text{ J}$. At the turning point $v = 0$ so $K_2 = 0$. Since $E = K_2 + U_2$, $2J = 0J + U_2 \Rightarrow U_2 = 2J$. This occurs at $x = 3$ m.

25.7. **(a)** The capacitor charge remains constant. No charge flows off the capacitor plates so by conservation of charge $Q_1 = Q_2$.

(b) The electric field strength also remains constant: $E = \dfrac{\eta}{\varepsilon_0} = \dfrac{Q/A}{\varepsilon_0}$. Q, A, and ε_0 are all constant.

25.8. $V_a = V_b > V_c > V_d = V_e$. $V = E_s$ where s is measured from the negative plate. The electric field E is constant inside a parallel-plate capacitor.

25.9. **(a)** $\dfrac{V_2}{V_1} = \dfrac{Es_2}{Es_1} = \dfrac{s_2}{s_1} = \dfrac{3\text{ mm}}{1\text{ mm}} = 3$

(b) $\dfrac{E_2}{E_1} = 1$. The field inside a parallel-plate capacitor is a constant: $E = \dfrac{\eta}{\varepsilon_0}$.

25.10. **(a)** $\dfrac{V_2}{V_1} = \dfrac{\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_2}}{\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_1}} = \dfrac{r_1}{r_2} = \dfrac{1\text{ mm}}{3\text{ mm}} = \dfrac{1}{3}$

(b) $\dfrac{E_2}{E_1} = \dfrac{\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_2^2}}{\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_1^2}} = \dfrac{r_1^2}{r_2^2} = \dfrac{(1\text{ mm})^2}{(3\text{ mm})^2} = \dfrac{1}{9}$

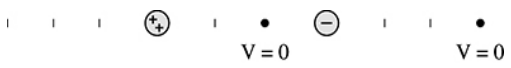
25.11. **(a)** $V_b > V_a = V_c$

(b) $V_a > V_b > V_c$

25.12. The potential due to the two charges at a position x is

$$U(x) = \frac{1}{4\pi\varepsilon_0}\frac{2q}{|x|} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{|x-3|}$$

where the $2q$ charge is at $x = 0$ and the $-q$ charge is at $x = 3$. Setting $U(x) = 0$ and solving for x yields the two locations.



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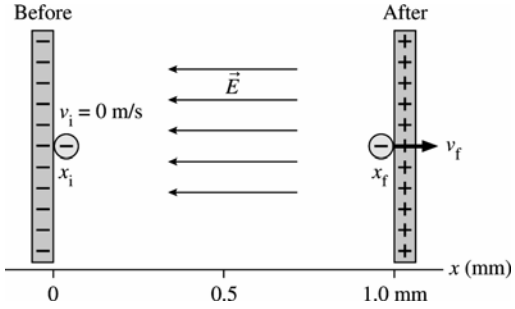
Exercises and Problems

Exercises

Section 25.1 Electric Potential Energy

25.1. Model: The mechanical energy of the electron is conserved. A parallel-plate capacitor has a uniform electric field.
Visualize:



The figure shows the before-and-after pictorial representation. The electron has an initial speed $v_i = 0$ m/s and a final speed v_f after traveling a distance $d = 1.0$ mm.

Solve: The electron loses potential energy and gains kinetic energy as it moves toward the positive plate. The potential energy U is defined as $U = U_0 + qEx$, where x is the distance from the negative plate and U_0 is the potential energy at the negative plate (at $x = 0$ m). Thus, the change in the potential energy of the electron is

$$\Delta U_e = U_f - U_i = (U_0 + qEd) - (U_0 + 0\text{ J}) = qEd$$

The change in the kinetic energy of the electron is

$$\Delta K = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2$$

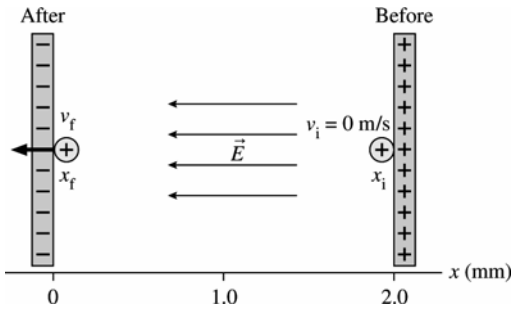
Now, the law of conservation of mechanical energy gives $\Delta K + \Delta U = 0$ J. This means

$$\tfrac{1}{2}mv_f^2 + qEd = 0\text{ J}$$

$$\Rightarrow v_f = \sqrt{\frac{-2qEd}{m}} = \sqrt{\frac{(-2)(-1.60\times10^{-19}\text{ C})(20{,}000\text{ N/C})(1.0\times10^{-3}\text{ m})}{9.11\times10^{-31}\text{ kg}}} = 2.7\times10^{6}\text{ m/s}$$

Assess: Note that $\Delta U_e = qEd$ is the change in the potential energy of the electron. It is negative because $q = -e$ for the electron. Thus, the potential energy becomes more negative as d increases, that is, the potential energy of the electron decreases with an increase in d (or x).

25.2. Model: The mechanical energy of the proton is conserved. A parallel-plate capacitor has a uniform electric field.
Visualize:



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The figure shows the before-and-after pictorial representation. The proton has an initial speed $v_i = 0$ m/s and a final speed v_f after traveling a distance $d = 2.0$ mm.

Solve: The proton loses potential energy and gains kinetic energy as it moves toward the negative plate. The potential energy U is defined as $U = U_0 + qEx$, where x is the distance from the negative plate and U_0 is the potential energy at the negative plate (at $x = 0$ m). Thus, the change in the potential energy of the proton is

$$\Delta U_p = U_f - U_i = (U_0 + 0\text{ J}) - (U_0 + qEd) = -qEd$$

The change in the kinetic energy of the proton is

$$\Delta K = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2.$$

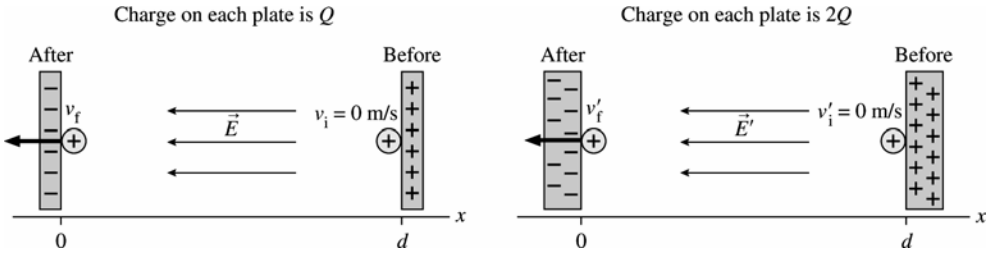
The law of conservation of energy is $\Delta K + \Delta U_p = 0$ J. This means

$$\tfrac{1}{2}mv_f^2 + (-qEd) = 0\text{ J}$$

$$\Rightarrow v_f = \sqrt{\frac{2qEd}{m}} = \sqrt{\frac{2(+1.60\times10^{-19}\text{ C})(50{,}000\text{ N/C})(2.0\times10^{-3}\text{ m})}{(1.67\times10^{-27}\text{ kg})}} = 1.4\times10^{5}\text{ m/s}$$

Assess: As described in Section 25.1, the potential energy for a charge q in an electric field E is $U = U_0 + qEx$, where x is the distance measured from the negative plate. Having $U = U_0$ at the negative plate (with $x = 0$ m) is completely arbitrary. We could have taken it to be zero. Note that only ΔU, and not U, has physical consequences.

25.3. Model: The mechanical energy of the proton is conserved. A parallel-plate capacitor has a uniform electric field.
Visualize:



The figure shows the before-and-after pictorial representation.

Solve: The proton loses potential energy and gains kinetic energy as it moves toward the negative plate. The potential energy is defined as $U = U_0 + qEx$, where x is the distance from the negative plate and U_0 is the potential energy at the negative plate (at $x = 0$ m). Thus, the change in the potential energy of the proton is

$$\Delta U_p = U_f - U_i = (U_0 + 0\text{ J}) - (U_0 + qEd) = -qEd$$

The change in the kinetic energy of the proton is

$$\Delta K = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2$$

Applying the law of conservation of energy $\Delta K + \Delta U_p = 0$ J, we have for the proton

$$\tfrac{1}{2}mv_f^2 + (-qEd) = 0\text{ J} \Rightarrow v_f^2 = \frac{2qEd}{m}$$

The same would be true for an electron; the only difference is the mass.

$$\frac{v_{fe}^2}{v_{fp}^2} = \frac{\dfrac{2qEd}{m_e}}{\dfrac{2qEd}{m_p}} \Rightarrow v_{fe}^2 = v_{fp}^2\frac{m_p}{m_e} = (50{,}000\text{ m/s})^2\frac{1.67\times10^{-27}\text{ kg}}{9.11\times10^{-31}\text{ kg}} \Rightarrow v_{fe} = 2.1\times10^{6}\text{ m/s}$$

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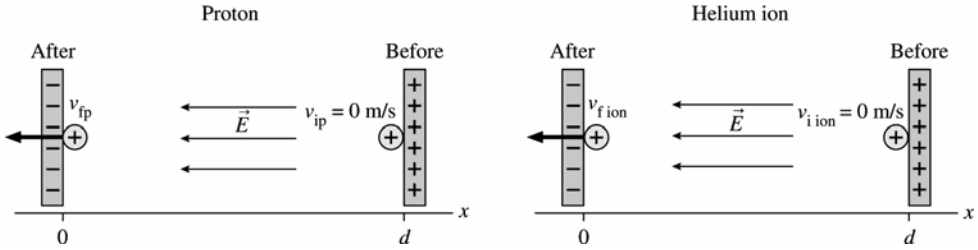
Assess: The electron is less massive and so accelerates more under the same force.

25.4. Model: The mechanical energy of the charged particles is conserved. A parallel-plate capacitor has a uniform electric field.
Visualize:



The figure shows the before-and-after pictorial representation.
Solve: The potential energy is defined as $U = U_0 + qEx$, where x is the distance from the negative plate and U_0 is the potential energy at the negative plate (at $x = 0$ m). Thus, the change in the potential energy of the proton as it moves from the positive plate to the negative plate is

$$\Delta U_p = U_f - U_i = (U_0 + 0\text{ J}) - (U_0 + eEd) = -eEd$$

This decrease in potential energy appears as an increase in the proton's kinetic energy:

$$\Delta K = K_f - K_i = \tfrac{1}{2}m_p v_{f\,p}^2 - \tfrac{1}{2}m_p v_{i\,p}^2 = \tfrac{1}{2}m_p v_{f\,p}^2$$

Applying the law of conservation of mechanical energy $\Delta K + \Delta U_p = 0$ J, we have

$$\tfrac{1}{2}m v_{f\,p}^2 + (-eEd) = 0\text{ J} \Rightarrow v_{f\,p}^2 = \frac{2eEd}{m_p}$$

When the proton is replaced with a helium ion and the same experiment is repeated,

$$v_{f\,ion}^2 = \frac{2eEd}{m_{ion}}$$

Dividing the two equations,

$$v_{f\,ion} = \sqrt{\frac{m_p}{m_{ion}}}\,v_{f\,p} = \sqrt{\tfrac{1}{4}}(50{,}000\text{ m/s}) = 25{,}000\text{ m/s}$$

Assess: Being a heavier particle, the helium ion's velocity is expected to be smaller compared to the proton's velocity.

Section 25.2 The Potential Energy of Point Charges

25.5. Model: The charges are point charges.
Solve: The electric potential energy of the proton is

$$\begin{aligned} U_{proton} &= U_{13} + U_{23} \\ &= (9.0\times10^9\text{ N m}^2/\text{C}^2)\left[\frac{(1.60\times10^{-19}\text{ C})(-1.60\times10^{-19}\text{ C})}{\sqrt{(2.0\times10^{-9}\text{ m})^2 + (0.50\times10^{-9}\text{ m})^2}} + \frac{(1.60\times10^{-19}\text{ C})(-1.60\times10^{-19}\text{ C})}{\sqrt{(2.0\times10^{-9}\text{ m})^2 + (0.50\times10^{-9}\text{ m})^2}}\right] \\ &= -1.12\times10^{-19}\text{ J} - 1.12\times10^{-19}\text{ J} = -2.24\times10^{-19}\text{ J} \approx -2.2\times10^{-19}\text{ J} \end{aligned}$$

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25.6. Model: The charges are point charges.
Solve: For a system of point charges, the potential energy is the sum of the potential energies due to all pairs of charges:

$$U_{\text{elec}} = \sum_{\text{i,j}} \frac{Kq_{\text{i}}q_{\text{j}}}{r_{\text{ij}}} = U_{12} + U_{13} + U_{23} = K\frac{q_1q_2}{r_{12}} + K\frac{q_1q_3}{r_{13}} + K\frac{q_2q_3}{r_{23}}$$

$$= (9.0\times10^9 \text{ N m}^2/\text{C}^2)\left[\frac{(1.0 \text{ nC})(-2.0 \text{ nC})}{0.030 \text{ m}} + \frac{(1.0 \text{ nC})(-2.0 \text{ nC})}{0.030 \text{ m}} + \frac{(-2.0 \text{ nC})(-2.0 \text{ nC})}{0.030 \text{ m}}\right]$$

$$= -0.60\times10^{-6} \text{ J} - 0.60\times10^{-6} \text{ J} + 1.20\times10^{-6} \text{ J} = 0 \text{ J}$$

Assess: Note that $U_{12} = U_{21}$, $U_{13} = U_{31}$, and $U_{23} = U_{32}$.

25.7. Model: The charges are point charges.
Solve: For a system of point charges, the potential energy is the sum of the potential energies due to all distinct pairs of charges:

$$U_{\text{elec}} = \sum_{\text{i,j}} \frac{Kq_{\text{i}}q_{\text{j}}}{r_{\text{ij}}} = U_{12} + U_{13} + U_{23}$$

$$= (9.0\times10^9 \text{ N m}^2/\text{C}^2)\left[\frac{(-4.0\times10^{-9} \text{ C})(3.0\times10^{-9} \text{ C})}{0.030 \text{ m}} + \frac{(-4.0\times10^{-9} \text{ C})(3.0\times10^{-9} \text{ C})}{0.040 \text{ m}} + \frac{(3.0\times10^{-9} \text{ C})(3.0\times10^{-9} \text{ C})}{\sqrt{(0.030 \text{ m})^2 + (0.040 \text{ m})^2}}\right]$$

$$= -3.60\times10^{-6} \text{ J} - 2.7\times10^{-6} \text{ J} + 1.62\times10^{-6} \text{ J} = -4.7\times10^{-6} \text{ J}$$

Assess: Note that $U_{12} = U_{21}$, $U_{13} = U_{31}$, and $U_{23} = U_{32}$.

25.8. Solve: Let the two unknown, positive charges be Q_1 and Q_2. They are separated by a distance $r_{12} = 5.0\times10^{-2}$ m. Their electric potential energy is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{12}} = 72\times10^{-6} \text{ J} \Rightarrow Q_1Q_2 = \frac{(72\times10^{-6} \text{ J})(5.0\times10^{-2} \text{ m})}{9.0\times10^9 \text{ N m}^2/\text{C}^2} = 4.0\times10^{-16} \text{ C}^2$$

The electric force between these two charges is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{12}^2} = (9.0\times10^9 \text{ N m}^2/\text{C}^2)\frac{4.0\times10^{-16} \text{ C}^2}{(5.0\times10^{-2} \text{ m})^2} = 1.44\times10^{-3} \text{ N}$$

Assess: As far as the magnitudes are concerned,

$$F = \frac{U}{r} = \frac{72\times10^{-6} \text{ J}}{5.0\times10^{-2} \text{ m}} = 1.4\times10^{-3} \text{ N}$$

Section 25.3 The Potential Energy of a Dipole

25.9. Model: A water molecule is at the point of minimum potential energy when it is aligned with an electric field. However, an external force can rotate the water molecule causing its dipole moment to make an angle with the field.
Solve: The potential energy of an electric dipole moment in a uniform electric field is $U_{\text{dipole}} = -\vec{p}\cdot\vec{E} = -pE\cos\theta$. This means

$$U_{\text{dipole parallel}} = -pE\cos0° = -pE \qquad U_{\text{dipole perpendicular}} = -pE\cos90° = 0 \text{ J}$$

$$\Rightarrow U_{\text{dipole perpendicular}} - U_{\text{dipole parallel}} = 1.0\times10^{-21} \text{ J} = 0 \text{ J} - (-pE)$$

$$\Rightarrow E = \frac{1.0\times10^{-21} \text{ J}}{p} = \frac{1.0\times10^{-21} \text{ J}}{6.2\times10^{-30} \text{ C m}} = 1.61\times10^8 \text{ N/C}$$

Assess: Note that the units with E are J/C m. Because 1 J/C m = 1 N m/C m = 1 N/C, the units of E are N/C.

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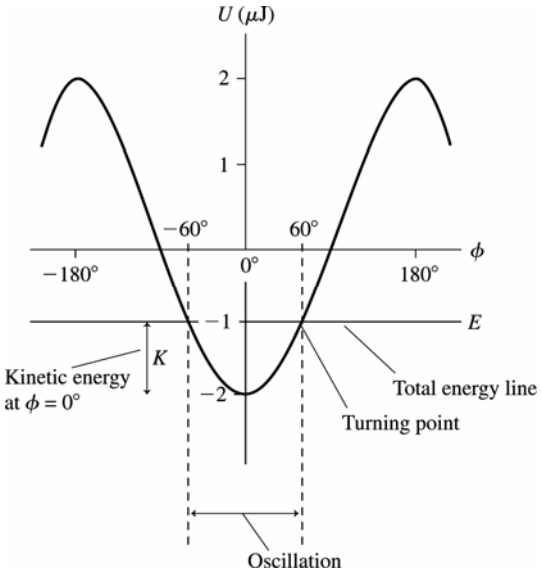
25.10. Model: An external electric field supplies energy to a dipole.
Visualize: On an energy diagram, the oscillation occurs between the points where the potential-energy curve crosses the total energy line.



Solve: (a) The potential energy of an electric dipole moment in a uniform electric field is $U = -\vec{p}\cdot\vec{E} = -pE\cos\phi$. This means

$$U_{\phi=0^\circ} = -pE = -2.0\ \mu\text{J} \qquad U_{\phi=60^\circ} = -pE\cos 60^\circ = -\tfrac{1}{2}pE = -\tfrac{1}{2}(2\ \mu\text{J}) = -1.0\ \mu\text{J}$$

The mechanical energy $E_{\text{mech}} = U + K$. We know that at $\phi = 60°$, $K_{\phi=60^\circ} = 0$ J. So,

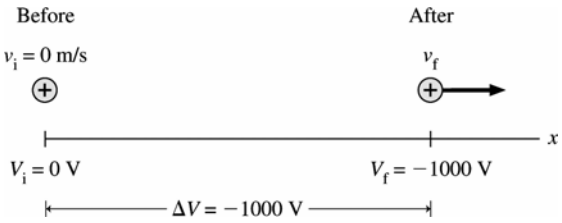
$$E_{\text{mech}} = U_{\phi=60^\circ} + K_{\phi=60^\circ} = -1.0\ \mu\text{J} + 0.0\ \text{J} = -1.0\ \mu\text{J}$$

(b) Conservation of mechanical energy gives

$$U_{\phi=60^\circ} + K_{\phi=60^\circ} = U_{\phi=0^\circ} + K_{t=0^\circ} \Rightarrow -1.0\ \mu\text{J} + 0.0\ \text{J} = -2.0\ \mu\text{J} + K_{\phi=0^\circ} \Rightarrow K_{\phi=0^\circ} = 1.0\ \mu\text{J}$$

Section 25.4 The Electric Potential

25.11. Model: Energy is conserved. The potential energy is determined by the electric potential.
Visualize:



The figure shows a before-and-after pictorial representation of a proton moving through a potential difference of −1000 V. A positive charge *speeds up* as it moves into a region of lower potential ($U \rightarrow K$).

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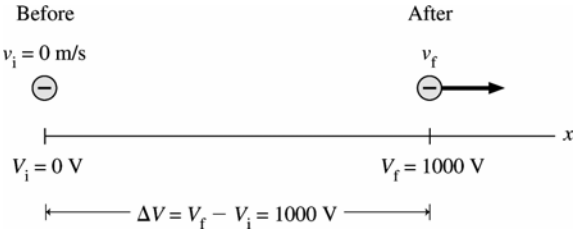
Solve: The potential energy of charge q is $U = qV$. Conservation of energy, expressed in terms of the electric potential V, is

$$K_f + qV_f = K_i + qV_i, \Rightarrow K_f = K_i + q(V_i - V_f) = 0\text{ J} - q\Delta V$$

$$\Rightarrow \tfrac{1}{2}mv_f^2 = -q(-1000\text{ V}) \Rightarrow v_f = \sqrt{\frac{2(1.60\times10^{-19}\text{ C})(1000\text{ V})}{1.67\times10^{-27}\text{ kg}}} = 4.38\times10^5\text{ m/s}$$

Assess: Note that the proton of our problem has nothing to do with creating the potential difference of –1000 V.

25.12. Model: Mechanical energy is conserved. The potential energy is determined by the electric potential.
Visualize:



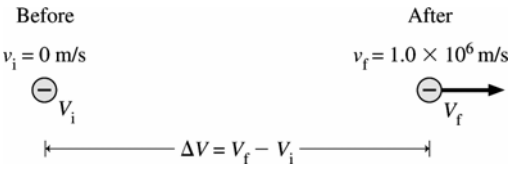
The figure shows a before-and-after pictorial representation of an electron moving through a potential difference of 1000 V. A negative charge *speeds up* as it moves into a region of higher potential ($U \rightarrow K$).
Solve: The potential energy of charge q is $U = qV$. Conservation of energy, expressed in terms of the electric potential V, is

$$K_f + qV_f = K_i + qV_i \Rightarrow K_f = K_i + q(V_i - V_f) = K_i - q(V_f - V_i)$$

$$\Rightarrow \tfrac{1}{2}mv_f^2 = 0\text{ J} - (-e)(\Delta V) \Rightarrow v_f = \sqrt{\frac{2e\Delta V}{m}} = \sqrt{\frac{2(1.60\times10^{-19}\text{ C})(1000\text{ V})}{9.11\times10^{-31}\text{ kg}}} = 1.87\times10^7\text{ m/s}$$

Assess: Note that the electric potential difference of 1000 V already existed in space due to other charges or sources. The electron of our problem has nothing to do with creating the potential.

25.13. Model: Energy is conserved. The potential energy is determined by the electric potential.
Visualize:



The figure shows a before-and-after pictorial representation of an electron moving through a potential difference. Because the negative electron gains speed as it travels, it moves into a region of higher potential ($U \rightarrow K$).
Solve: The potential energy of charge q is $U = qV$. Using the conservation of energy equation,

$$K_f + qV_f = K_i + qV_i \Rightarrow V_f - V_i = \Delta V = \frac{1}{q}(K_i - K_f) = \frac{1}{(-e)}(0\text{ J} - \tfrac{1}{2}mv_f^2)$$

$$\Rightarrow \Delta V = \frac{mv_f^2}{2e} = \frac{(9.11\times10^{-31}\text{ kg})(2.0\times10^6\text{ m/s})^2}{2(1.60\times10^{-19}\text{ C})} = 11.4\text{ V}$$

Assess: A positive value of ΔV shows that the electron moved from a region of lower potential to a region of higher potential.

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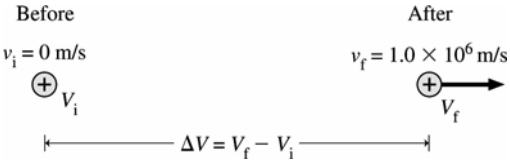
25.14. Model: Energy is conserved. The potential energy is determined by the electric potential.
Visualize:



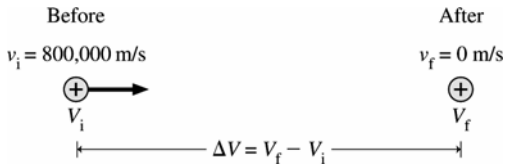
The figure shows a before-and-after pictorial representation of a He^+ ion moving through a potential difference. The ion's initial speed is zero and its final speed is 2.0×10^6 m/s. A positive charge *speeds up* as it moves into a region of lower potential ($U \rightarrow K$).
Solve: The potential energy of charge q is $U = qV$. Conservation of energy, expressed in terms of the electric potential V, is

$$K_f + qV_f = K_i + qV_i \Rightarrow q(V_f - V_i) = K_i - K_f$$

$$\Rightarrow \Delta V = \frac{K_i - K_f}{q} = \frac{0\text{ J} - \frac{1}{2}mv_f^2}{q} = -\frac{4(1.67\times10^{-27}\text{ kg})(2.0\times10^6\text{ m/s})^2}{2(1.60\times10^{-19}\text{ C})} = -8.4\times10^4\text{ V}$$

Assess: This result implies that the helium ion moves from a higher potential toward a lower potential.

25.15. Model: Energy is conserved. The potential energy is determined by the electric potential.
Visualize:



The figure shows a before-and-after pictorial representation of a proton moving through a potential difference.
Solve: (a) Because the proton is a positive charge and it slows down as it travels, it must be moving from a region of lower potential to a region of higher potential.
(b) Using the conservation of energy equation,

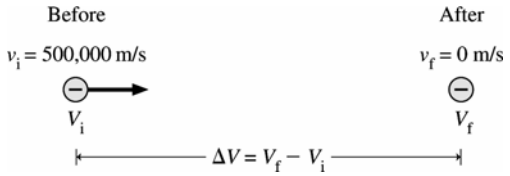
$$K_f + U_f = K_i + U_i \Rightarrow K_f + qV_f = K_i + qV_i$$

$$\Rightarrow V_f - V_i = \frac{1}{q}(K_i - K_f) = \frac{1}{(e)}\left(\tfrac{1}{2}mv_i^2 - 0\text{ J}\right)$$

$$\Rightarrow \Delta V = \frac{mv_i^2}{2e} = \frac{(1.67\times10^{-27}\text{ kg})(800{,}000\text{ m/s})^2}{2(1.60\times10^{-19}\text{ C})} = 3340\text{ V}$$

Assess: A positive ΔV confirms that the proton moves into a higher potential region.

25.16. Model: Energy is conserved. The potential energy is determined by the electric potential.
Visualize:



The figure shows a before-and-after pictorial representation of an electron moving through a potential difference.

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Solve: (a) Because the electron is a negative charge and it slows down as it travels, it must be moving from a region of higher potential to a region of lower potential.
(b) Using the conservation of energy equation,

$$K_f + U_f = K_i + U_i \Rightarrow K_f + qV_f = K_i + qV_i$$
$$\Rightarrow V_f - V_i = \frac{1}{q}(K_i - K_f) = \frac{1}{(-e)}\left(\tfrac{1}{2}mv_i^2 - 0\text{ J}\right)$$
$$\Rightarrow \Delta V = -\frac{mv_i^2}{2e} = -\frac{(9.11\times10^{-31}\text{ kg})(500{,}000\text{ m/s})^2}{2(1.60\times10^{-19}\text{ C})} = -0.712\text{ V}$$

Assess: The negative sign with ΔV verifies that the electron moves from a higher potential region to a lower potential region.

25.17. Model: The point charge is a particle in free fall.
Visualize: The initial speed of the proton is zero.
Solve: Use the kinematic equations to find the speed squared after free falling a distance Δy.

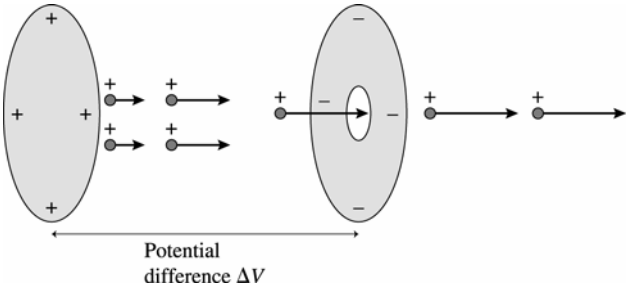
$$v_f^2 = v_i^2 + 2g\Delta y \Rightarrow v_f^2 = 2g\Delta y$$

Now employ the energy conservation equation

$$K_f + qV_f = K_i + qV_i \Rightarrow q\Delta V = \Delta K \Rightarrow$$
$$\Delta V = \frac{\Delta K}{q} = \frac{\frac{1}{2}mv_f^2}{q} = \frac{\frac{1}{2}m2g\Delta y}{q} = \frac{mg\Delta y}{q} = \frac{(1.67\times10^{-27}\text{ kg})(9.8\text{ m/s}^2)(100\text{ m})}{1.6\times10^{-19}\text{ C}} = 1.0\times10^{-5}\text{ V}$$

Assess: The units are J/C = V.

25.18. Visualize: From conservation of energy, the increase in kinetic energy of the protons equals their decrease in potential energy. In other words, $\Delta K = -\Delta U = -q\Delta V$. In the following figure, the source of the potential difference is a capacitor, with the negatively charged plate having a hole cut in the center to allow protons through. Notice that while the protons are between the plates, they are accelerating, but once they pass through the plate on the right, they maintain a constant speed. This is because there is no electric field outside an ideal capacitor.



Solve: If we solve the equation for q, we get:

$$q = \frac{-\Delta K}{\Delta V} = \frac{\Delta K}{|\Delta V|} = \frac{0.10\text{ J}}{10\times10^6\text{ V}} = 1.0\times10^{-8}\text{ C} = 10\text{ nC}$$

Here we have used the fact that since the protons are accelerating, they must be traveling through a decreasing potential. Consequently, ΔV is negative and $-\Delta V = |\Delta V|$.

Assess: Since the protons have a charge of e, their kinetic energy after passing the potential difference is $10\text{ MeV} = \left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)(10\times10^6\text{ eV}) = 1.60\times10^{-12}\text{ J}$. Since the total energy was 0.10 J, we can determine the

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number of protons, N: $N = (0.10\text{ J})/(1.60\times10^{-12}\text{ J}) = 6.3\times10^{10}$. As a check, the charge of this quantity of protons is $(6.3\times10^{10})(1.60\times10^{-19}\text{ C}) = 10\text{ nC}$, as we find in the statement of the problem.

25.19. Model: Assume the particles start at rest.
Visualize: The particles will have different final speeds because they have different masses.
Solve: After accelerating through a potential difference of 9.0 V each particle will have qV of kinetic energy.

$$q\Delta V = K = \tfrac{1}{2}mv_f^2 \Rightarrow v_f = \sqrt{\frac{2q\Delta V}{m}}$$

(a) Insert the mass of the proton.

$$v_f = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2(1.6\times10^{-19}\text{ C})(9.0\text{ V})}{1.67\times10^{-27}\text{ kg}}} = 4.2\times10^{3}\text{ m/s}$$

(b) Now insert the mass of the electron.

$$v_f = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2(1.6\times10^{-19}\text{ C})(9.0\text{ V})}{9.11\times10^{-31}\text{ kg}}} = 1.8\times10^{6}\text{ m/s}$$

Assess: The final speed for the electron is much greater than for the proton, as we expected.

25.20. Solve: (a) The charge on the electron is 1.0 e, and after accelerating through 1.0 V it will have a kinetic energy of 1.0 eV.

$$K = qV = (1.0\ e)(1.6\times10^{-19}\text{ C}/e)(1.0\text{ V}) = 1.6\times10^{-19}\text{ C}\cdot\text{V} = 1.6\times10^{-19}\text{ J}$$

(b) Now insert the mass of the electron.

$$K = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5000\text{ eV})(1.6\times10^{-19}\text{ J/eV})}{1.67\times10^{-27}\text{ kg}}} = 9.8\times10^{5}\text{ m/s}$$

Assess: The final speed for the proton is very large, but typical for this situation.

Section 25.5 The Electric Potential Inside a Parallel-Plate Capacitor

25.21. Solve: By definition 1 V = 1 J/C and 1 J = 1 N m. Consequently,

$$1\frac{\text{V}}{\text{m}} = 1\frac{\text{J}}{\text{C m}} = 1\frac{\text{N m}}{\text{C m}} = 1\frac{\text{N}}{\text{C}}$$

25.22. Model: The electric potential difference between the plates is determined by the uniform electric field in the parallel-plate capacitor.
Solve: (a) The potential of an ordinary AA or AAA battery is 1.5 V. Actually, this is the potential difference between the two terminals of the battery. If the electric potential of the negative terminal is taken to be zero, then the positive terminal is at a potential of 1.5 V.
(b) If a battery with a potential difference of 1.5 V is connected to a parallel-plate capacitor, the potential difference between the two capacitor plates is also 1.5 V. Thus,

$$\Delta V_C = 1.5\text{ V} = V_+ - V_- = Ed$$

where d is the separation between the two plates. The electric field inside a parallel-plate capacitor is

$$E = \frac{\eta}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \Rightarrow 1.5\text{ V} = \left(\frac{Q}{A\varepsilon_0}\right)d$$

$$\Rightarrow Q = \frac{(1.5\text{ V})(A\varepsilon_0)}{d} = \frac{(1.5\text{ V})(4.0\times10^{-2}\text{ m})^2(8.85\times10^{-12}\text{ C}^2/\text{Nm}^2)}{1.0\times10^{-3}\text{ m}} = 2.1\times10^{-11}\text{ C}$$

Thus, the battery moves 2.1×10^{-11} C of electron charge from the positive to the negative plate of the capacitor.

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25.23. Model: The electric potential difference between the plates is determined by the uniform electric field in the parallel-plate capacitor.

Solve: (a) The potential difference ΔV_C across a capacitor of spacing d is related to the electric field inside by

$$E = \frac{\Delta V_C}{d} \Rightarrow \Delta V_C = Ed = (1.0\times10^5 \text{ V/m})(0.0020 \text{ m}) = 200 \text{ V}$$

(b) The electric field of a capacitor is related to the surface charge density by

$$E = \frac{\eta}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0}$$

$$\Rightarrow Q = \varepsilon_0 AE = (8.85\times10^{-12} \text{ C}^2/\text{N m}^2)\pi(1.5\times10^{-2} \text{ m})^2(1.0\times10^5 \text{ V/m}) = 6.3\times10^{-10} \text{ C}$$

25.24. Model: The electric potential between the plates of a parallel-plate capacitor is determined by the uniform electric field between the plates.

Solve: (a) The potential difference across the plates of a capacitor is

$$\Delta V_C = Ed = \left(\frac{\eta}{\varepsilon_0}\right)d = \frac{(Q/A)}{\varepsilon_0}d = \frac{Qd}{A\varepsilon_0} = \frac{(0.708\times10^{-9} \text{ C})(1.00\times10^{-3} \text{ m})}{(4.00\times10^{-4} \text{ m}^2)(8.85\times10^{-12} \text{ C}^2/\text{N m}^2)} = 200 \text{ V}$$

(b) For $d = 2.00$ mm, $\Delta V_C = 400$ V.

Assess: Note that the units in part (a) are N m/C. But Exercise 25.22 showed that $1 \text{ N/C} = 1 \text{ V/m}$, so $1 \text{ N m/C} = 1 \text{ V}$. We also see that the potential difference across a parallel-plate capacitor is directly proportional to the plate separation.

25.25. Model: Energy is conserved. The electron's potential energy inside the capacitor can be found from the capacitor's electric potential.

Solve: (a) The voltage across the capacitor is

$$\Delta V_C = Ed = (5.0\times10^5 \text{ V/m})(2.0\times10^{-3} \text{ m}) = 1000 \text{ V}$$

(b) The electron has charge $q = -e$, and its potential energy at a point where the capacitor's potential is V is $U = -eV$. Since the electron is launched from the negative (lower potential) plate toward the positive (higher potential) plate, its potential energy becomes more negative (because of the negative sign of the electron charge). That is, the potential energy decreases, which must lead to an increase in the kinetic energy. Conversely, the electron's speed as it is launched is smaller than 2.0×10^7 m/s. The conservation of energy equation is

$$K_f + qV_f = K_i + qV_i \Rightarrow \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2 + q(V_f - V_i)$$

$$\Rightarrow v_i^2 = v_f^2 + \frac{2}{m}(-e)(1000 \text{ V})$$

$$\Rightarrow v_i = \sqrt{(2.0\times10^7 \text{ m/s})^2 - \frac{2(1.60\times10^{-19} \text{ C})(1000 \text{ V})}{9.11\times10^{-31} \text{ kg}}} = 7.0\times10^6 \text{ m/s}$$

25.26. Model: Energy is conserved. The proton's potential energy inside the capacitor can be found from the capacitor's potential difference.

Solve: (a) The electric potential at the midpoint of the capacitor is 250 V. This is because the potential inside a parallel-plate capacitor is $V = Es$ where s is the distance from the negative electron. The proton has charge $q = e$ and its potential energy at a point where the capacitor's potential is V is $U = eV$. The proton will gain potential energy

$$\Delta U = e\Delta V = e(250 \text{ V}) = 1.60\times10^{-19} \text{ C}(250 \text{ V}) = 4.00\times10^{-17} \text{ J}$$

if it moves all the way to the positive plate. This increase in potential energy comes at the expense of kinetic energy which is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.67\times10^{-27} \text{ kg})(200{,}000 \text{ m/s})^2 = 3.34\times10^{-17} \text{ J}$$

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This available kinetic energy is not enough to provide for the increase in potential energy if the proton is to reach the positive plate. Thus the proton does not reach the plate because $K < \Delta U$.

(b) The energy-conservation equation $K_f + U_f = K_i + U_i$ is

$$\tfrac{1}{2}mv_f^2 + qV_f = \tfrac{1}{2}mv_i^2 + qV_i \Rightarrow \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_i^2 + q(V_i - V_f)$$

$$\Rightarrow v_f = \sqrt{v_i^2 + \frac{2q}{m}(V_i - V_f)} = \sqrt{(2.0\times10^5\ \text{m/s})^2 + \frac{2(1.60\times10^{-19}\ \text{C})(250\ \text{V} - 0\ \text{V})}{1.67\times10^{-27}\ \text{kg}}} = 2.96\times10^5\ \text{m/s}$$

Section 25.6 The Electric Potential of a Point Charge

25.27. Model: The charge is a point charge.

Solve: (a) The electric potential of the point charge q is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = (9.0\times10^9\ \text{N m}^2/\text{C}^2)\left(\frac{2.0\times10^{-9}\ \text{C}}{r}\right) = \frac{18.0\ \text{N m}^2/\text{C}}{r}$$

For points A and B, $r = 0.010$ m. Thus,

$$V_A = V_B = \frac{18.0\ \text{N m}^2/\text{C}}{0.010\ \text{m}} = 1800\,\frac{\text{Nm}}{\text{C}} = 1800\left(\frac{\text{V}}{\text{m}}\right)\text{m} = 1.80\ \text{kV}$$

For point C, $r = 0.020$ m and $V_C = 900$ V.

(b) The potential differences are

$$\Delta V_{AB} = V_B - V_A = 1.80\ \text{kV} - 1.80\ \text{kV} = 0\ \text{V} \qquad \Delta V_{BC} = V_C - V_B = 0.90\ \text{kV} - 1.80\ \text{kV} = -0.90\ \text{kV}$$

25.28. Solve: Outside a charged sphere of radius R, the electric potential is identical to that of a point charge Q at the center. That is,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \qquad r \ge R$$

The potential of the ball bearing is the potential right on the surface of the ball bearing. Thus,

$$V = \frac{(9.0\times10^9\ \text{N m}^2/\text{C}^2)(2.0\times10^9)(-1.60\times10^{-19}\ \text{C})}{0.50\times10^{-3}\ \text{m}} = -5.8\ \text{kV}$$

25.29. Model: Outside a charged sphere the electric potential is identical to that of a point charge at the center.

Solve: (a) For a proton, assumed to be a point charge, the electric potential is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{(+e)}{r} = (9.0\times10^9\ \text{N m}^2/\text{C}^2)\frac{1.60\times10^{-19}\ \text{C}}{0.053\times10^{-9}\ \text{m}} = 27\ \text{V}$$

(b) The potential energy of a charge q at a point where the potential is V is $U = qV$. The potential energy of the electron in the proton's potential is

$$U = (-1.60\times10^{-19}\ \text{C})(27\ \text{V}) = -4.3\times10^{-18}\ \text{J}$$

Section 25.7 The Electric Potential of Many Charges

25.30. Model: The net potential is the sum of the potentials due to each charge.

Solve: The potential at the dot is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$$

$$= (9.0\times10^9\ \text{N m}^2/\text{C}^2)\left[\frac{2.0\times10^{-9}\ \text{C}}{0.040\ \text{m}} + \frac{-1.0\times10^{-9}\ \text{C}}{0.050\ \text{m}} + \frac{2.0\times10^{-9}\ \text{C}}{0.030\ \text{m}}\right] = 870\ \text{V}$$

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Assess: Potential is a scalar quantity, so we found the net potential by adding three scalar quantities.

25.31. Model: The net potential is the sum of the potentials due to each charge.
Solve: From the geometry in the figure,

$$\frac{1.5\text{ cm}}{r_1}=\frac{1.5\text{ cm}}{r_2}=\frac{1.5\text{ cm}}{r_3}=\cos 30^\circ \Rightarrow r_1=r_2=r_3=\frac{1.5\text{ cm}}{\cos 30^\circ}=1.732\text{ cm}$$

The potential at the dot is

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}+\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}+\frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$$
$$=(9.0\times10^9\text{ N m}^2/\text{C}^2)\left[\frac{1.0\times10^{-9}\text{ C}}{0.01732\text{ m}}-\frac{2.0\times10^{-9}\text{ C}}{0.01732\text{ m}}-\frac{2.0\times10^{-9}\text{ C}}{0.01732\text{ m}}\right]=-1600\text{ V}$$

Assess: Potential is a scalar quantity, so we found the net potential by adding three scalar quantities.

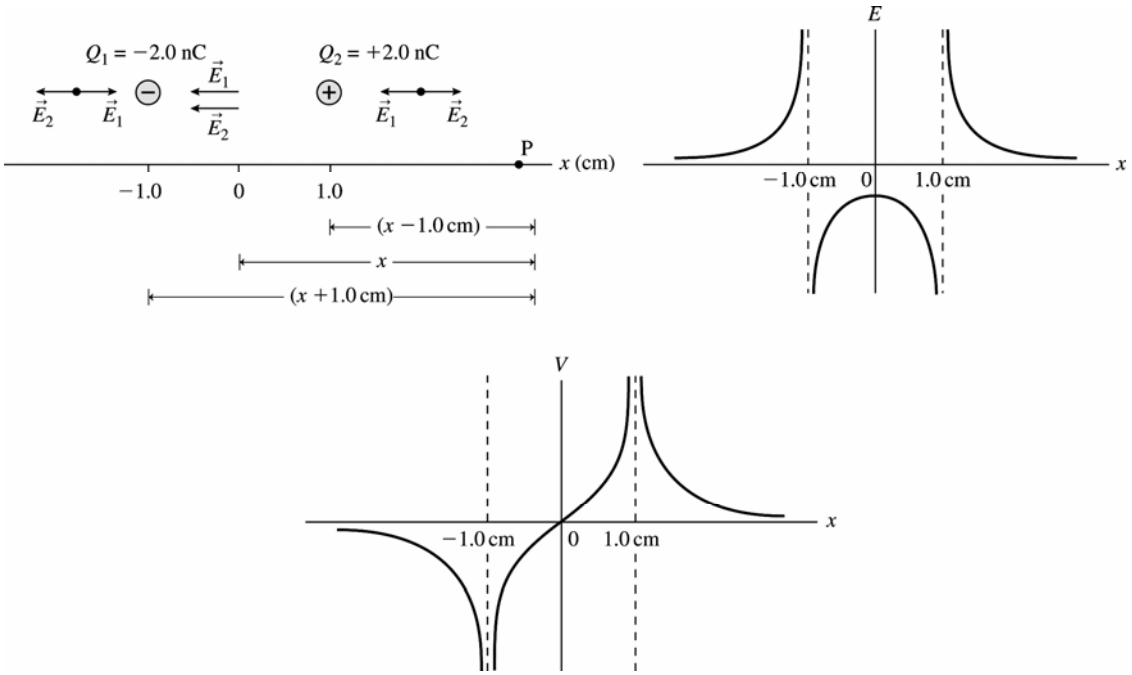
25.32. Model: The electric potential at the dot is the sum of the potentials due to each charge.
Solve: The electric potential at the dot is

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}+\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}+\frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$$
$$=(9.0\times10^9\text{ Nm}^2/\text{C}^2)\left[\frac{q}{(0.020\text{ m})^2+(0.040\text{ m})^2}+\frac{-5.0\times10^{-9}\text{ C}}{0.040\text{ m}}+\frac{5.0\times10^{-9}\text{ C}}{0.020\text{ m}}\right]=3140\text{ V}$$

Solving yields $q=1.00\times10^{-8}\text{ C}=10\text{ nC}$.

Assess: Potential is a scalar quantity, so we found the net potential by adding three scalar quantities.

25.33. Model: While the net potential is the sum of the scalar potentials due to each charge, the net electric field is the vector sum of the electric fields.
Visualize:



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Solve: (a) Let the two charges be labeled as $Q_1 = -2.0$ nC and $Q_2 = +2.0$ nC. As pictorially shown in the figure, the net electric field will not become zero anywhere except at infinity. We will now show this result mathematically. Let the point of zero electric field be at a distance x from the origin. Then,

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 = 0 \text{ N/C} \Rightarrow E_1 = -E_2 \Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{2.0\times10^{-9}\text{ C}}{(x+1.0\text{ cm})^2} = \frac{1}{4\pi\varepsilon_0}\frac{2.0\times10^{-9}\text{ C}}{(x-1.0\text{ cm})^2}$$

$$\Rightarrow (x+1.0\text{ cm})^2 = (x-1.0\text{ cm})^2$$

For $x > 0$, this equation can only be satisfied if $x = \infty$. That is, the electric field is not zero except at $+\infty$ and at $-\infty$.
(b) Because potential is a scalar quantity, the potential of charge Q_1 is always negative and the potential of charge Q_2 is always positive. These two scalar numbers will add to give zero potential at $x = 0$ m and at $\pm\infty$. Mathematically this can be shown as follows. When $-1.0\text{ cm} \le x \le 1.0$ cm, let the point of zero potential be at x. Then, the condition $V_1 + V_2 = 0$ V/m is

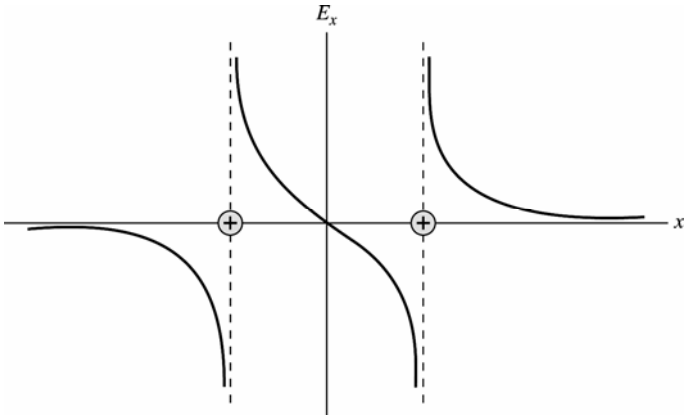
$$\frac{1}{4\pi\varepsilon_0}\frac{(-2\times10^{-9}\text{ C})}{1.0\text{ cm}+x} + \frac{1}{4\pi\varepsilon_0}\frac{(2\times10^{-9}\text{ C})}{(1.0\text{ cm}-x)} = 0 \Rightarrow -(1.0\text{ cm}+x)+(1.0\text{ cm}-x) = 0 \Rightarrow x = 0\text{ cm}$$

When $x > 1.0$ cm,

$$\frac{1}{4\pi\varepsilon_0}\frac{(-2\times10^{-9}\text{ C})}{(x+1.0\text{ cm})} + \frac{1}{4\pi\varepsilon_0}\frac{(2\times10^{-9}\text{ C})}{(x-1.0\text{ cm})} = 0 \Rightarrow (x+1.0\text{ cm}) = (x-1.0\text{ cm})$$

This equation can only be satisfied at $x = \pm\infty$.
(c) The graphs are shown in the figure above.

25.34. Model: While the potential is the sum of the scalar potentials due to each charge, the electric field is the vector sum of the electric fields due to each charge.
Solve: (a) As V is always positive, both charges must be positive. The positive signs for the two equal charges at $x = a$ and at $x = b$ are also consistent with the behavior of the potential in the range $a < x < b$.
(b) By the symmetry of the drawing about the middle, we infer that the magnitudes of the charges are the same. Thus $|q_a/q_b| = 1$.
(c)



The graph of E_x, the x-component of the electric field, as a function of x is shown in the figure.

25.35. Model: Consider the rod to be a line of charge.
Visualize: Divide the rod into small segments, each with charge $\pm\Delta q$.

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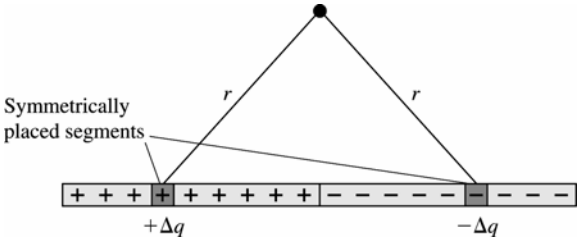

Solve: Consider two segments, one positive and one negative, equally distant from the center of the rod. These segments are the same distance *r* from the dot. Thus the contribution of this pair of segments to the potential at the dot is

$$V_+ + V_- = \frac{1}{4\pi\varepsilon_0}\frac{+\Delta q}{r} + \frac{1}{4\pi\varepsilon_0}\frac{-\Delta q}{r} = 0 \text{ V}$$

Since we can divide the entire rod into pairs of symmetrically placed segments, the net result of adding the potentials due to each pair is $V = 0$ V.

Assess: This conclusion depends on the dot being directly outward from the midpoint of the rod. The potential is *not* zero at other points.

25.36. Model: The ball of charge behaves as a point charge. Assume the potential energy of the system is zero when the electron is far away. Removing one electron does not appreciably change the charge on the ball.

Visualize: $W_{\text{elec}} = \Delta U$. The two charges are $r = 2.5$ cm apart. The charge on the ball is $Q_{\text{ball}} = \eta A = (10\ \mu\text{C/m}^2)4\pi(2.5\text{ cm})^2 = 78.5$ nC.

Solve: Since $U_{\text{f}} = 0$ then $W_{\text{elect}} = U_{\text{i}}$.

$$U_{\text{i}} = \frac{Kq_1q_2}{r} = \frac{Kq_{\text{ball}}q_{\text{elec}}}{r} = \frac{K(q_{\text{ball}})(-e)}{r} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(78.5\text{ nC})(-1.6\times10^{-19}\ \text{C})}{0.025\text{ m}} = -4.5\times10^{-15}\ \text{J}$$

Assess: The work is negative because a negative electron would be pushed away from a positive ball.

Problems

25.37. Solve: Let the unknown charges be Q_1 and Q_2. Then $Q_1 + Q_2 = 30\times10^{-9}$ C and

$$U = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{2.0\times10^{-2}\text{ m}} = -180\times10^{-6}\ \text{J}$$

$$\Rightarrow Q_1Q_2 = \frac{-(180\times10^{-6}\ \text{J})(2.0\times10^{-2}\ \text{m})}{9.0\times10^9\ \text{N m}^2/\text{C}^2} = -4.0\times10^{-16}\ \text{C}^2$$

Solving the first equation for Q_2 and substituting into the second equation,

$$Q_1(30\times10^{-9}\ \text{C} - Q_1) = -4.0\times10^{-16}\ \text{C}^2 \Rightarrow Q_1^2 - (30\times10^{-9}\ \text{C})Q_1 - (4.0\times10^{-16}\ \text{C}^2) = 0$$

$$\Rightarrow Q_1 = \frac{(30\times10^{-9}\ \text{C}) \pm \sqrt{(-30\times10^{-9}\ \text{C})^2 + 4(4.0\times10^{-16}\ \text{C}^2)}}{2} \Rightarrow Q_1 = +40\text{ nC and } -10\text{ nC}$$

That is, the two charges are –10 nC and 40 nC.

Assess: As they must, the two charges when added yield a total charge of 30 nC, and when substituted into the potential energy equation yield $U = -180\times10^{-6}$ J.

25.38. Model: While the net potential is the sum of the potentials due to each charge, the net electric field is the vector sum of the electric fields.

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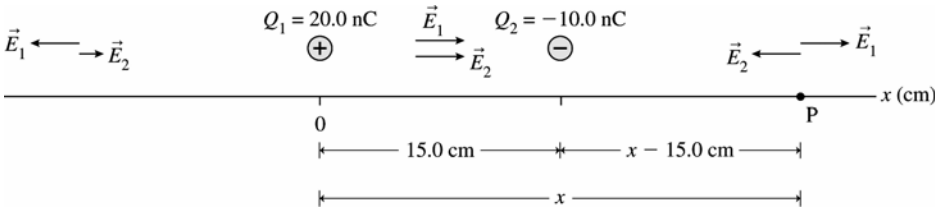
Visualize:



The charge $Q_1 = 20.0$ nC is at the origin. The charge $Q_2 = -10.0$ nC is 15.0 cm to the right of the charge Q_1 on the *x*-axis.

Solve: (a) As the pictorial representation shows, the point P on the *x*-axis where the electric field is zero can only be on the right side of the charge Q_2, that is, at $x \geq 15.0$ cm. At this point $E_1 = E_2$, so we have

$$\frac{1}{4\pi\varepsilon_0}\frac{20.0\times10^{-9}\text{ C}}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{10.0\times10^{-9}\text{ C}}{(x-15.0\text{ cm})^2} \Rightarrow x^2 = 2(x-15.0\text{ cm})^2$$

$$\Rightarrow x^2 - (60.0\text{ cm})x + 450\text{ cm}^2 = 0 \Rightarrow x = \frac{(60.0\text{ cm}) \pm \sqrt{3600\text{ cm}^2 - 1800\text{ cm}^2}}{2}$$

$$\Rightarrow x = 51.2\text{ cm and } 8.8\text{ cm}$$

The root $x = 8.8$ cm is not possible physically. So, the electric fields cancel out at $x = 51.2$ cm. The electric potential at this point is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_2} = (9.0\times10^9\text{ N m}^2/\text{C}^2)\left[\frac{20.0\times10^{-9}\text{ C}}{0.512\text{ m}} + \frac{-10.0\times10^{-9}\text{ C}}{(0.512\text{ m} - 0.150\text{ m})}\right] = +103\text{ V}$$

(b) The point on the *x*-axis where the potential is zero can be obtained from the condition $V_1 + V_2 = 0$ V, which is

$$\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_2} = 0\text{ V} \Rightarrow \frac{20.0\times10^{-9}\text{ C}}{x} + \frac{-10.0\times10^{-9}\text{ C}}{(15.0\text{ cm} - x)} = 0$$

$$\Rightarrow 2(15.0\text{ cm} - x) - x = 0 \Rightarrow x = 10.0\text{ cm}$$

The electric field 10.0 cm away from charge Q_1 is

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{20.0\times10^{-9}\text{ C}}{(0.100\text{ m})^2}\hat{i} + \frac{1}{4\pi\varepsilon_0}\frac{(10.0\times10^{-9}\text{ C})}{(0.050\text{ m})^2}\hat{i} = 5.40\times10^4\hat{i}\text{ V/m}$$

The magnitude is 5.40×10^4 V/m.

25.39. Model: The net potential is the sum of the scalar potentials due to each charge.
Visualize:

Solve: Let the point on the *x*-axis where the electric potential is zero be at a distance *x* from the origin. At this point, $V_1 + V_2 = 0$ V. This means

$$\frac{1}{4\pi\varepsilon_0}\left\{\frac{3.0\times10^{-9}\text{ C}}{x} + \frac{-1.0\times10^{-9}\text{ C}}{|x - 4.0\text{ cm}|}\right\} = 0\text{ V} \Rightarrow -x + 3|x - 4.0\text{ cm}| = 0\text{ cm}$$

Either $-x + 3(x - 4.0\text{ cm}) = 0$ cm, or $-x + 3(4.0\text{ cm} - x) = 0$ cm. In the first case, $x = 6.0$ cm and, in the second case, $x = 3.0$ cm. That is, we have two points on the *x*-axis where the potential is zero.

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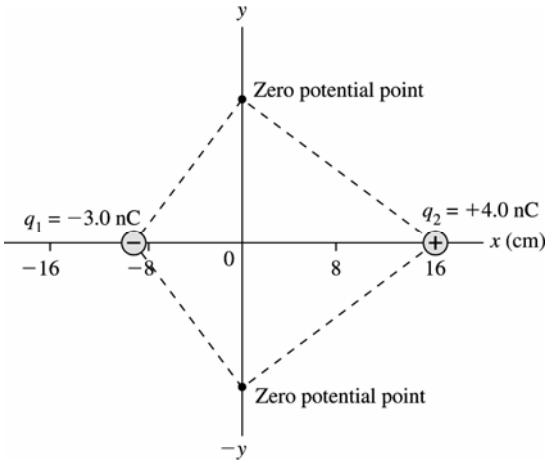
25.40. Model: The net potential is the sum of the scalar potentials due to each charge.
Visualize:



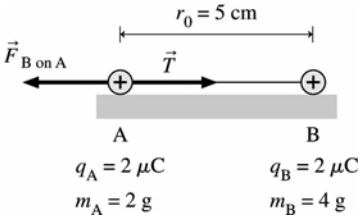
Solve: Let the point on the *y*-axis where the electric potential is zero be at a distance *y* from the origin. At this point, $V_1 + V_2 = 0$ V. This means

$$\frac{1}{4\pi\varepsilon_0}\left\{\frac{q_1}{r_2}+\frac{q_2}{r_2}\right\} = 0 \text{ V} \Rightarrow \frac{-3.0\times10^{-9}\text{ C}}{\sqrt{(-9.0\text{ cm})^2+y^2}} + \frac{4.0\times10^{-9}\text{ C}}{\sqrt{(16.0\text{ cm})^2+y^2}} = 0$$

$$\Rightarrow 3\sqrt{(16\text{ cm})^2+y^2} = 4\sqrt{(-9\text{ cm})^2+y^2} \Rightarrow 9(256\text{ cm}^2+y^2) = 16(81\text{ cm}^2+y^2)$$

$$\Rightarrow 7y^2 = 1008\text{ cm}^2 \Rightarrow y = \pm 12\text{ cm}$$

25.41. Model: Mechanical energy is conserved. The charged metal cubes can be modeled as point charges.
Visualize:



Solve: (a) The system could have both kinetic and potential energy, although here $K = 0$ J. The energy of the system is

$$E_0 = K_0 + U_0 = 0\text{ J} + \frac{q_A q_B}{4\pi\varepsilon_0 r_0} = \frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(2.0\times10^{-6}\text{ C})(2.0\times10^{-6}\text{ C})}{0.050\text{ m}} = 0.72\text{ J}$$

(b) In static equilibrium, the net force on cube A is zero. Thus

$$T = F_{\text{B on A}} = \frac{|q_A||q_B|}{4\pi\varepsilon_0 r_0^2} = \frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(2.0\times10^{-6}\text{ C})(2.0\times10^{-6}\text{ C})}{(0.050\text{ m})^2} = 14\text{ N}$$

(c) The cubes move apart due to the repulsive electric force between them. The surface is frictionless, so they continue to slide without stopping. When they are very far apart $(r_1 \to \infty)$, their potential energy is $U_1 \to 0$ J. Energy is conserved, so we have

$$E_1 = K_1 + U_1 = \tfrac{1}{2}m_A v_{A1}^2 + \tfrac{1}{2}m_B v_{B1}^2 + 0\text{ J} = E_0$$

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Momentum is also conserved: $P_{\text{after}} = m_A v_{A1} + m_B v_{B1} = P_{\text{before}} = 0$ kg m/s. Note that these are *velocities* and that v_{A1} is a negative number. From the momentum equation,

$$v_{A1} = -\frac{m_B v_{B1}}{m_A}$$

Substituting this into the energy equation,

$$E_0 = \tfrac{1}{2} m_A \left(-\frac{m_B v_{B1}}{m_A} \right)^2 + \tfrac{1}{2} m_B v_{B1}^2 = \tfrac{1}{2} m_B \left(\frac{m_B}{m_A} + 1 \right) v_{B1}^2$$

$$\Rightarrow v_{B1} = \sqrt{\frac{2E_0}{m_B(m_B/m_A + 1)}} = \sqrt{\frac{2(0.720 \text{ J})}{(0.004 \text{ kg})(4 \text{ g}/2 \text{ g} + 1)}} = 10.95 \text{ m/s}$$

Using this result, we can then find $v_{A1} = -m_B v_{B1}/m_A = -21.9$ m/s. These are the velocities, so the final *speeds* are 22 m/s for the 2 g cube and 11 m/s for the 4 g cube.

25.42. Model: Energy is conserved. The charged spheres are point charges.
Visualize: Label the spheres 1, 2, 3, and 4 in a clockwise manner, with the sphere in the upper left-hand corner being sphere 1.
Solve: The total potential energy of the four spheres before they are allowed to move is

$$U_i = (U_{12} + U_{23} + U_{34} + U_{41}) + (U_{13} + U_{24})$$

All the charges are identical, $r_{12} = r_{23} = r_{34} = r_{41} = 1.0$ cm, and $r_{13} = r_{24} = \sqrt{2.0}$ cm. The initial potential energy is

$$U_i = 4\left(\frac{1}{4\pi e_0} \frac{(10 \times 10^{-9} \text{ C})(10 \times 10^{-9} \text{ C})}{0.010 \text{ m}} \right) + 2\left(\frac{1}{4\pi e_0} \frac{(10 \times 10^{-9} \text{ C})(10 \times 10^{-9} \text{ C})}{0.01414 \text{ m}} \right) = 48.73 \times 10^{-5} \text{ J}$$

Since all charges are at rest, $K_i = 0$ J. As the spheres are allowed to move away from one another and they are far apart, $U_f = 0$ J. The final kinetic energy is

$$K_f = 4\left(\tfrac{1}{2} m v_f^2 \right) = 2(1.0 \times 10^{-3} \text{ kg}) v_f^2$$

From the energy conservation equation $U_f + K_f = U_i + K_i$,

$$2(1.0 \times 10^{-3} \text{ kg}) v_f^2 = 48.73 \times 10^{-5} \text{ J} \Rightarrow v_f = 0.49 \text{ m/s}$$

25.43. Model: Energy is conserved. The potential energy is determined by the electric potential.
Solve: The proton at point A is at a potential of 30 V and its speed is 50,000 m/s. At point B, the proton is at a potential of −10 V, and we are asked to find its speed. Clearly, the proton moves into a lower potential region, so its speed will increase. The conservation of energy equation $K_f + U_f = K_i + U_i$ is

$$\tfrac{1}{2} m v_f^2 + (+e)(-10 \text{ V}) = \tfrac{1}{2} m (50{,}000 \text{ m/s})^2 + (+e)(30 \text{ V})$$

$$\Rightarrow v_f = \sqrt{(50{,}000 \text{ m/s})^2 + \frac{2(1.60 \times 10^{-19} \text{ C})(40 \text{ V})}{1.67 \times 10^{-27} \text{ kg}}} = 1.0 \times 10^5 \text{ m/s}$$

Assess: The speed of the proton is higher, as expected.

25.44. Solve:
(a) The work that must be done to move a charged particle through a potential difference is $q\Delta V$.

$$W = q\Delta V = (1.6 \times 10^{-19} \text{ C})(70 \text{ mV}) = 1.1 \times 10^{-20} \text{ J}$$

(b) The body uses 20% of 100 J of energy every second to pump sodium ions. We will round the answer to part (a) so we won't need a calculator to get our one-significant-figure answer.

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$$\text{number of sodium ions} = \frac{2\times10^{1}\ \text{J}}{1\times10^{-20}\ \text{J/ion}} = 2\times10^{21}\ \text{ions}$$

Assess: The answer is a huge number, and even if we divide by the number of cells in our body there are still many sodium ions pumped each second in every cell.

25.45. Model: Energy is conserved.
Solve: Turning points at ±8.0 cm mean that $x_{\max} = \pm 8.0$ cm. At the turning points, the energy is all potential energy and the kinetic energy is zero. The mechanical energy of the charged particle is

$$E = U + K = U_{\max} + 0\ \text{J} = 5000\ q(x_{\max})^2 = 5000(10.0\times10^{-9}\ \text{C})(0.080\ \text{m})^2 = 3.2\times10^{-7}\ \text{J}$$

The conservation of energy equation $K_1 + U_1 = K_2 + U_2$ is

$$K_{\max} + 0\ \text{J} = 0\ \text{J} + U_{\max} \Rightarrow \tfrac{1}{2}mv_{\max}^2 = 3.20\times10^{-7}\ \text{J} \Rightarrow v_{\max} = \sqrt{\frac{2(3.20\times10^{-7}\ \text{J})}{(1.0\times10^{-3}\ \text{kg})}} = 2.5\ \text{cm/s}$$

25.46. Model: The proton is a point charge.
Visualize: We are given $V(x) = V_0\sin(2\pi x/\lambda)$, where $V_0 = 5000$ V and $\lambda = 1.0$ mm, so $V = 0$ V at $x = 0$ m. To move down the *x*-axis we must give it enough speed to get over the potential hill that is 5000 V high, since the sine function has a maximum value of 1.
Solve: (a) The proton will need qV in energy to get over the hill.

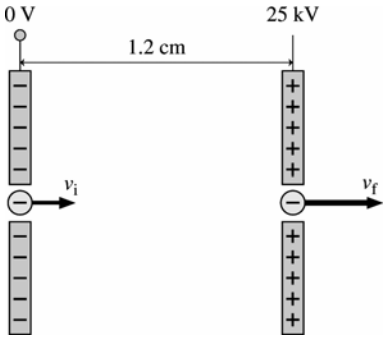
$$K_f + qV_f = K_i + qV_i \Rightarrow K_i = qV_f = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2qV_f}{m}} = \sqrt{\frac{2(1.6\times10^{-19}\ \text{C})(5000\ \text{V})}{1.67\times10^{-27}\ \text{kg}}} = 9.8\times10^{5}\ \text{m/s}$$

(b) The maximum speed will be when the proton is in the bottom of a trough of the potential function. If the proton has a speed of 9.8×10^{6} m/s at $x = 0$, then it will have a speed of 0 m/s at the top of the hill. It then falls 10,000 V in potential to the bottom (not just back to zero, but to –5000 V).

$$v = \sqrt{\frac{2qV_f}{m}} = \sqrt{\frac{2(1.6\times10^{-19}\ \text{C})(10000\ \text{V})}{1.67\times10^{-27}\ \text{kg}}} = 1.4\times10^{6}\ \text{m/s}$$

Assess: The maximum speed is $\sqrt{2}$ times the speed at the origin.

25.47. Model: Energy is conserved.
Visualize:



Solve: (a) The electric field inside a parallel-plate capacitor is constant with strength

$$E = \frac{\Delta V}{d} = \frac{(25\times10^{3}\ \text{V})}{0.012\ \text{m}} = 2.1\times10^{6}\ \text{V/m}$$

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(b) Assuming the initial velocity is zero, energy conservation yields

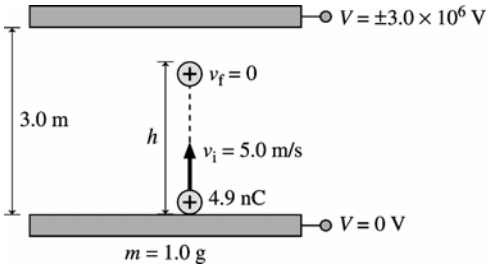
$$U_i = K_f + U_f$$

$$0 = \frac{1}{2}m_e v_f^2 + (-e)(Ed)$$

$$\Rightarrow v_f = \sqrt{\frac{2(1.60\times10^{-19}\text{ C})(2.1\times10^{6}\text{ V/m})(0.012\text{ m})}{9.11\times10^{-31}\text{ kg}}} = 9.4\times10^{7}\text{ m/s}$$

Assess: This speed is about 31% the speed of light. At that speed, relativity should be taken into account.

25.48. Model: Mechanical energy is conserved.
Visualize:



Solve: (a) Both gravitational and electric potential energy are involved. The electric potential between the plates is $V = Es$, where s is measured from the more negative plate. The electric field between the plates is

$$E = \frac{\Delta V}{d} = \frac{3.0\times10^{6}\text{ V}}{3.0\text{ m}} = 1.00\times10^{6}\text{ V/m}$$

Conservation of energy is used to find the height.

$$K_i + U_{gi} + U_{ei} = K_f + U_{gf} + U_{ef}$$

$$\frac{1}{2}mv_i^2 + 0\text{ J} + q(0\text{ V}) = 0\text{ J} + mgh + q(Eh)$$

$$\Rightarrow h = \frac{\frac{1}{2}mv_i^2}{mg + qE} = \frac{\frac{1}{2}(1.0\times10^{-3}\text{ kg})(5.0\text{ m/s})^2}{(1.0\times10^{-3}\text{ kg})(9.8\text{ m/s}^2) + (4.0\times10^{-9}\text{ C})(1.00\times10^{6}\text{ V/m})} = 0.85\text{ m}$$

(b) The top plate is at a negative potential, so the charge is attracted to it. The electric potential is now zero at the top plate and positive at the bottom plate. Does the charge make it all the way to the top plate? If we assume it does not and obtain a height more than 3.0 m then we will know that it does.
Conservation of energy for the charge requires

$$\frac{1}{2}mv_i^2 + 0\text{ J} + qE(3.0\text{ m}) = mgh + qE(3.0\text{ m} - \text{h}) \Rightarrow \frac{1}{2}mv_i^2 = (mg - qE)$$

$$\Rightarrow h = \frac{\frac{1}{2}mv_i^2}{mg - qE} = 2.6\text{ m}$$

Assess: Since $h < 3.0$ m the charge did not hit the top plate.

25.49. Model: Model the cube as a point charge. The potential falls away as $1/r$ from the sphere.
Visualize: Because of the inverse relationship between potential and distance the potential where the cube starts is 150,000 V.

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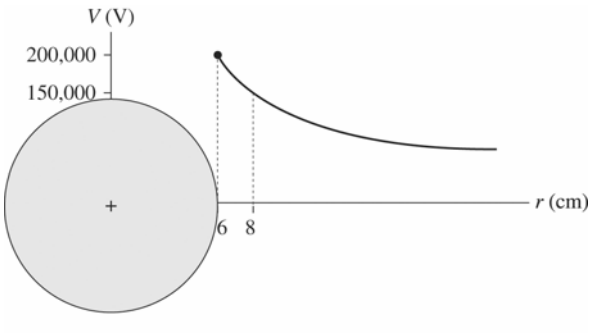


Solve: Use the energy conservation equation.

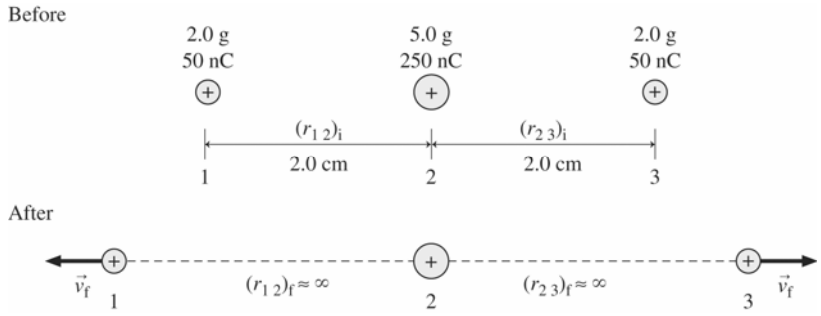
$$K_f + qV_f = K_i + qV_i \Rightarrow K_f = qV_i \Rightarrow \tfrac{1}{2}mv^2 = qV_i \Rightarrow q = \frac{\frac{1}{2}mv^2}{V_i} = \frac{\frac{1}{2}(0.0050\text{ kg})(3.0\text{ m})^2}{150{,}000\text{ V}} = 1.5\times10^{-7}\text{ C} = 150\text{ nC}$$

While the charge on the cube might be reasonable, 3.0 m/s is not a very fast projectile speed, and the charge would have to be greater to have a faster projectile speed, so this is not a practical projectile launcher.

Assess: The wording of the problem suggests this is not a practical projectile launcher, as we verified.

25.50. Model: Model the buttons as point charges. Since all the buttons are positive, they will push each other away, but due to symmetry the bigger button in the middle will not move.

Visualize:



Solve: (a) There are three different interactions that have a potential energy: U_{12}, U_{13}, U_{23}.

(b) Use the energy conservation equation. Due to blessed symmetry both buttons 1 and 3 end up with the same speed, and since they have the same mass we can add their two kinetic energies together.

$$K_f + U_f = K_i + U_i \Rightarrow \tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2 + 0 = \frac{Kq_1q_2}{(r_{12})_i} + \frac{Kq_1q_3}{(r_{13})_i} + \frac{Kq_2q_3}{(r_{23})_i}$$

$$2\left(\tfrac{1}{2}mv^2\right) = K\left[\frac{q_1q_2}{(r_{12})_i} + \frac{q_1q_3}{(r_{13})_i} + \frac{q_2q_3}{(r_{23})_i}\right] \Rightarrow v = \sqrt{\frac{K}{m}\left[\frac{q_1q_2}{(r_{12})_i} + \frac{q_1q_3}{(r_{13})_i} + \frac{q_2q_3}{(r_{23})_i}\right]}$$

$$= \sqrt{\frac{9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}{0.0020\text{ kg}}\left[\frac{(50\text{ nC})(250\text{ nC})}{0.020\text{ m}} + \frac{(50\text{ nC})(50\text{ nC})}{0.040\text{ m}} + \frac{(50\text{ nC})(250\text{ nC})}{0.020\text{ m}}\right]} = 2.4\text{ m/s}$$

The two buttons with mass 2.0 g and charge 50 nC each move away with speed 2.4 m/s while the middle larger button does not move at all.

Assess: The speeds seem reasonable.

25.51. Model: Energy is conserved. The electron ends up so far away from the glass sphere that we can consider its potential energy to be zero.

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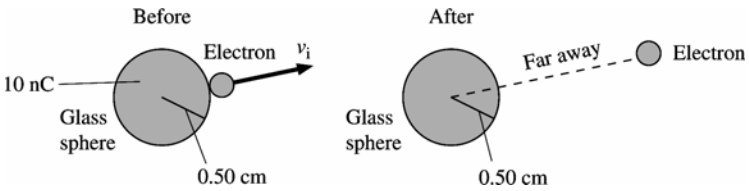
Visualize:



The minimum speed to escape is the speed that allows the electron to reach $r_f = \infty$ when $v_f = 0$ m/s.

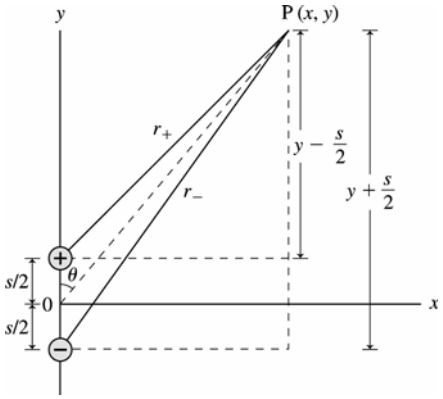
Solve: The conservation of energy equation $K_f + U_f = K_i + U_i$ is

$$0\text{ J} + 0\text{ J} = \tfrac{1}{2}mv_i^2 + qV_i \Rightarrow 0\text{ J} = \tfrac{1}{2}mv_i^2 + (-e)\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{R}\right)$$

$$\Rightarrow v_i = \sqrt{\frac{2e}{m}\frac{1}{4\pi\varepsilon_0}\frac{q}{R}} = \sqrt{\frac{2(1.60\times10^{-19}\text{ C})}{9.11\times10^{-31}\text{ kg}}(9.0\times10^9\text{ N m}^2/\text{C}^2)\left(\frac{10\times10^{-9}\text{ C}}{0.50\times10^{-2}\text{ m}}\right)} = 8.0\times10^7\text{ m/s}$$

Assess: This speed is so fast that relativity should be used.

25.52. Model: The charges making the dipole are point charges. The potential is the sum of potentials from each charge.
Visualize:



Point P is located at a distance r from the center of the dipole and it makes an angle θ with the dipole axis.

Solve: The potential at P due to the dipole is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_+} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{r_-} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{x^2 + (y - s/2)^2}} - \frac{1}{\sqrt{x^2 + (y + s/2)^2}}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{x^2 + y^2 - ys + (s/2)^2}} - \frac{1}{\sqrt{x^2 + y^2 + ys + (s/2)^2}}\right]$$

Because $x, y >> s$, $(s/2)^2$ can be ignored compared with x^2, y^2, and ys. The potential is

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{r^2 - ys}} - \frac{1}{\sqrt{r^2 + ys}}\right] = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1 - \frac{ys}{r^2}\right)^{-1/2} - \frac{1}{r}\left(1 + \frac{ys}{r^2}\right)^{-1/2}\right]$$

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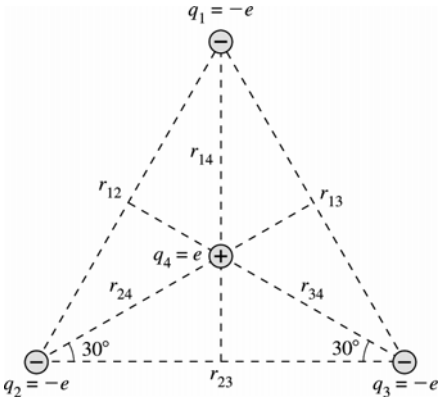
Using the binomial expansion,

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r}\left[\left(1+\frac{ys}{2r^2}+\cdots\right)-\left(1-\frac{ys}{2r^2}+\cdots\right)\right] = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r}\frac{2ys}{2r^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{(Qs)y}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{(Qs)r\cos\theta}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

where $p = Qs$ is the dipole moment of the dipole.

25.53. Model: The electrons and the proton are point charges.
Visualize:



Solve: We are given that $r_{12} = r_{23} = r_{13} = 1.0\times10^{-9}$ m. From the geometry of the figure,

$$\frac{\frac{1}{2}r_{23}}{r_{24}} = \cos 30^\circ \Rightarrow r_{24} = \frac{r_{23}}{2\cos 30^\circ} = 0.5774\times10^{-9}\ \text{m} = r_{14} = r_{34}$$

The contributions to the total potential energy are

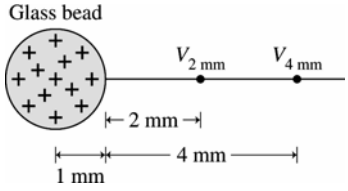
$$U_{12} = U_{13} = U_{23} = \frac{(9.0\times10^9\ \text{N m}^2/\text{C}^2)(-1.60\times10^{-19}\ \text{C})(-1.60\times10^{-19}\ \text{C})}{1.0\times10^{-9}\ \text{m}} = 2.304\times10^{-19}\ \text{J}$$
$$U_{14} = U_{24} = U_{34} = \frac{(9.0\times10^9\ \text{N m}^2/\text{C}^2)(-1.60\times10^{-19}\ \text{C})(1.60\times10^{-19}\ \text{C})}{0.5774\times10^{-9}\ \text{m}} = -3.990\times10^{-19}\ \text{J}$$

Summing all of the contributions,

$$U_{\text{elec}} = U_{12}+U_{13}+U_{23}+U_{14}+U_{24}+U_{34}$$
$$= 3(2.304\times10^{-19}\ \text{J}) + 3(-3.990\times10^{-19}\ \text{J}) = -5.1\times10^{-19}\ \text{J}$$

Assess: Note that $U_{12} = U_{21}$, $U_{13} = U_{31}$, and $U_{23} = U_{32}$, $U_{14} = U_{41}$, $U_{24} = U_{42}$, and $U_{34} = U_{43}$.

25.54. Model: Outside a charged sphere the electric potential is identical to that of a point charge at the center.
Visualize:



Solve: For $r \geq R$, $V = Q/4\pi\varepsilon_0 r$. The potentials at the two points are

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$$V_{2\text{ mm}} = \frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)Q}{(1\text{ mm}+2\text{ mm})} = \frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)Q}{3.0\times10^{-3}\text{ m}} \qquad V_{4\text{ mm}} = \frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)Q}{5.0\times10^{-3}\text{ m}}$$

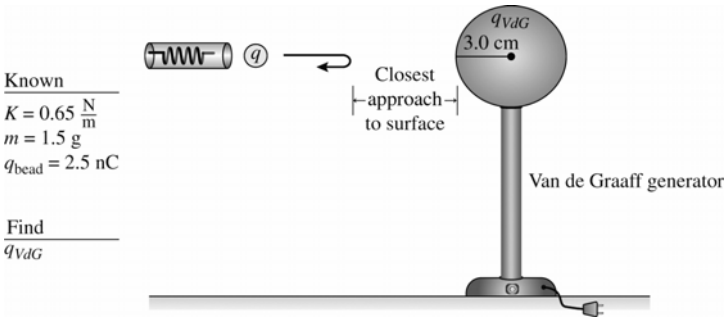
$$\Rightarrow V_{2\text{ mm}} - V_{4\text{ mm}} = +500\text{ V} = (9.0\times10^9\text{ N m}^2/\text{C}^2)Q\left(\frac{1}{3.0\times10^{-3}\text{ m}} - \frac{1}{5.0\times10^{-3}\text{ m}}\right)$$

$$\Rightarrow Q = \frac{500\text{ V}}{(9.0\times10^9\text{ N m}^2/\text{C}^2)}\left(\frac{15.0\times10^{-6}\text{ m}^2}{2.0\times10^{-3}\text{ m}}\right) = 4.2\times10^{-10}\text{ C}$$

Assess: Do not forget to include the radius of the glass bead in *r*.

25.55. Model: Ignore gravitational effects and consider the motion as one-dimensional in the *x*-direction. Model the bead as a point charge. As indicated, we assume the bead starts far enough away that the initial electric potential energy is negligible.

Visualize: Because of spherical symmetry we can consider the charge on the Van de Graaff generator to be concentrated at its center.



Solve: Use conservation of energy. The spring potential energy before the bead is fired equals the electric potential energy at closest approach (at both times the kinetic energy is zero).

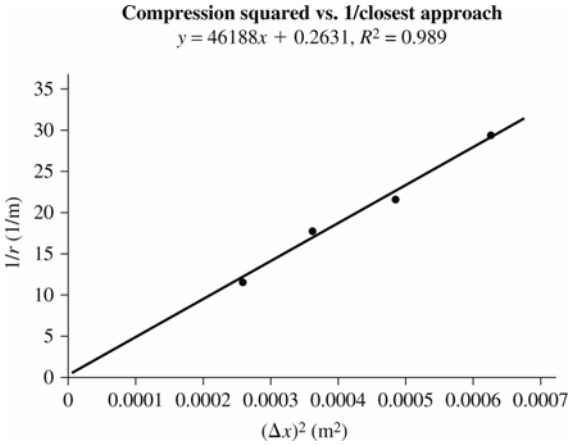
$$U_{\text{sp}} = U_{\text{elec}}$$

$$\frac{1}{2}k(\Delta x)^2 = K\frac{q_1q_2}{r}$$

The closest approach *r* must be measured from the center of the Van de Graaff sphere so we add 3.0 cm to each measurement of the closest approach. Rearrange the equation to produce

$$\frac{1}{r} = \left(\frac{k}{2}\frac{1}{K(q_{\text{bead}})(q_{VdG})}\right)(\Delta x)^2$$

This leads us to believe that a graph of $\frac{1}{r}$ vs. $(\Delta x)^2$ would produce a straight line whose slope is $\dfrac{k}{2K(q_{\text{bead}})(q_{VdG})}$ and whose intercept is zero.



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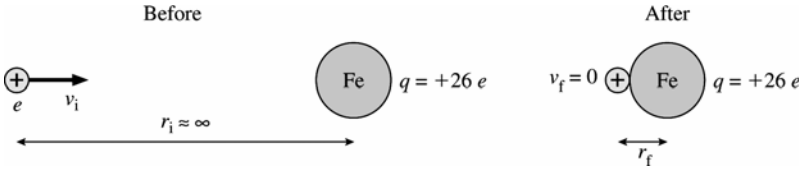
We see that the linear fit is good and that the slope is $46188\ \text{m}^{-3}$.

$$q_{VdG} = \frac{k}{2K(q_{\text{bead}})(\text{slope})} = \frac{0.65\ \text{N/m}}{2(9.0\times10^{9}\ \text{N m}^2/\text{C}^2)(2.5\ \text{nC})(46188\ \text{m}^{-3})} = 310\ \text{nC}$$

Assess: The intercept of our graph is very small, as we expect. The units cancel properly. The mass of the bead was irrelevant.

25.56. Model: Energy is conserved. Because the iron nucleus is very large compared to the proton, we will assume that the nucleus does not move (no recoil) and that the proton is essentially a point particle with no diameter.
Visualize:



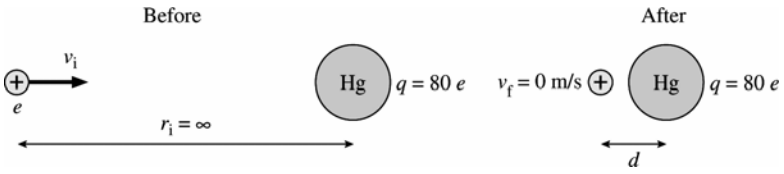
The proton is fired from a distance much greater than the nuclear diameter, so $r_i \approx \infty$ and $U_i \approx 0$ J. Because the nucleus is so small, a proton that is even a few atoms away is, for all practical purposes, at infinity. As the proton approaches the nucleus, it is slowed by the repulsive electric force. At the end point, the proton has just reached the surface of the nucleus (r_f = nuclear diameter) with $v_f = 0$ m/s. (The proton won't remain at this point but will be pushed back out again, but the subsequent motion is not part of this problem.)
Solve: Initially, the proton has kinetic energy but no potential energy. At the point of closest approach, where $v_f = 0$ m/s, the proton has potential energy but no kinetic energy. Energy is conserved, so $K_f + U_f = K_i + U_i$. This equation is

$$0\ \text{J} + \frac{(e)(26e)}{4\pi\varepsilon_0 r_f} = \tfrac{1}{2}m_{\text{proton}}v_i^2 + 0\ \text{J}$$

where r_f is half the nuclear diameter. The initial speed of the proton is

$$v_i = \sqrt{\frac{2(e)(26e)}{4\pi\varepsilon_0 r_f m_{\text{proton}}}} = \sqrt{\frac{2(1.6\times10^{-19}\ \text{C})(26\times1.6\times10^{-19}\ \text{C})(9.0\times10^{9}\ \text{N m}^2/\text{C}^2)}{(1.67\times10^{-27}\ \text{kg})(4.5\times10^{-15}\ \text{m})}} = 4.0\times10^{7}\ \text{m/s}$$

25.57. Model: Energy is conserved. Because the mercury nucleus is very large compared to the proton, we will assume that the nucleus does not move (no recoil) and that the proton is essentially a point particle with no diameter.
Visualize:



The proton is fired from a distance much greater than the nuclear diameter, so $r_i = \infty$ and $U_i = 0$ J. Because the nucleus is so small, a proton that is even a few atoms away is, for all practical purposes, at infinity. As the proton approaches the nucleus, it is slowed by the repulsive electric force. At the end point of the problem, the proton reaches the distance of closest approach (d) with $v_f = 0$ m/s. (The proton won't remain at this point but will be pushed back out again, but the subsequent motion is not part of the problem.)

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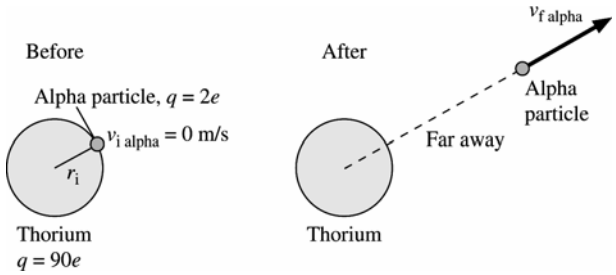
Solve: Initially, the proton has kinetic energy but no potential energy. At the point of closest approach, where $v_f = 0$ m/s, the proton has potential energy but no kinetic energy. Energy is conserved, so $K_f + U_f = K_i + U_i$. This equation is

$$0\text{ J} + \frac{e(80e)}{4\pi\varepsilon_0 d} = \tfrac{1}{2}m_{\text{proton}}v_i^2 + 0\text{ J}$$

$$\Rightarrow d = \frac{160e^2}{4\pi\varepsilon_0 m_{\text{proton}}v_i^2} = \frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(160)(1.6\times10^{-19}\text{ C})^2}{(1.67\times10^{-27}\text{ kg})(4.0\times10^7\text{ m/s})^2} = 1.38\times10^{-14}\text{ m} = 13.8\text{ fm}$$

The radius of the nucleus is 7.0 fm, so the proton's closest approach to the surface is 6.8 fm.

25.58. Model: Energy is conserved.
Visualize:



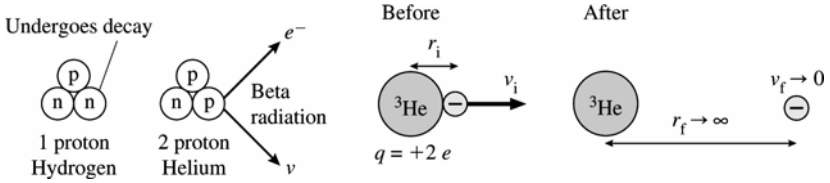
The alpha particle is initially at rest $(v_{\text{i alpha}} = 0\text{ m/s})$ at the surface of the thorium nucleus. The potential energy of the alpha particle is $U_{\text{i alpha}}$. After the decay, the alpha particle is far away from the thorium nucleus, $U_{\text{f alpha}} = 0$ J, and moving with speed $v_{\text{f alpha}}$.

Solve: Initially, the alpha particle has potential energy and no kinetic energy. As the alpha particle is detected in the laboratory, the alpha particle has kinetic energy but no potential energy. Energy is conserved, so $K_{\text{f alpha}} + U_{\text{f alpha}} = K_{\text{i alpha}} + U_{\text{i alpha}}$. This equation is

$$\tfrac{1}{2}mv_{\text{f alpha}}^2 + 0\text{ J} = 0\text{ J} + \frac{1}{4\pi\varepsilon_0}\frac{(2e)(90e)}{r_i}$$

$$\Rightarrow v_{\text{f alpha}} = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{(360e^2)}{mr_i}} = \sqrt{\frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)360(1.60\times10^{-19}\text{ C})^2}{4(1.67\times10^{-27}\text{ kg})(7.5\times10^{-15}\text{ m})}} = 4.1\times10^7\text{ m/s}$$

25.59. Model: Energy is conserved.
Visualize:



Solve: (a) The neutron has zero charge. In the decay $\text{n} \rightarrow \text{p}^+ + e^- + \nu$, where the neutrino ν is neutral, the net charge of the final state is $e + (-e) + 0 = 0$. So charge *is* conserved.

(b) As shown in the figure, the tritium nucleus has one proton and two neutrons. It is the single proton that makes this an isotope of hydrogen, the $Z = 1$ element in the periodic table. One of the neutrons undergoes the decay $\text{n} \rightarrow \text{p} + \text{e} + \nu$.

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The electron and neutrino are ejected from the nucleus as beta radiation, but the new proton remains in the nucleus. The nucleus still has three nucleons, but now two are protons and only one is a neutron. An atom whose nucleus contains two protons is the $Z = 2$ element of the periodic table, namely helium.

(c) As shown in the figure, the electron is "launched" from the surface of the ^{3}He nucleus with speed v_i. The initial energy is

$$E_i = K_i + U_i = \tfrac{1}{2}m_e v_i^2 + \frac{q_{elec}q_{nuc}}{4\pi\varepsilon_0 r_i} = \tfrac{1}{2}m_e v_i^2 + \frac{(-e)(2e)}{4\pi\varepsilon_0 r_i}$$

If the electron just barely escapes, it slows to $v_f \to 0$ m/s as $r_f \to \infty$. Its final energy is $K_f = K_f + U_f = 0\text{ J} + 0\text{ J} = 0\text{ J}$. Energy is conserved, $E_f = E_i$, so

$$\tfrac{1}{2}m_e v_i^2 + \frac{(-e)(2e)}{4\pi\varepsilon_0 r_i} = 0\text{ J}$$

$$\Rightarrow v_i = \sqrt{-\frac{2}{m_e}\frac{(-e)(2e)}{4\pi\varepsilon_0 r_i}} = \sqrt{\frac{(2)(9.0\times10^9\text{ N m}^2/\text{C}^2)(2)(1.60\times10^{-19}\text{ C})^2}{(9.11\times10^{-31}\text{ kg})(1.5\times10^{-15}\text{ m})}} = 8.21\times10^8\text{ m/s}$$

Assess: Our "classical" analysis gives an answer that exceeds the speed of light $(3\times10^8$ m/s), which is forbidden by Einstein's theory of relativity. We would need to perform a relativistic analysis, which is beyond the scope of this chapter, to get the correct minimum escape speed.

25.60. Model: The electric field inside a capacitor is uniform.

Solve: (a) Because the parallel-plate capacitor was connected to the terminals of a 15 V battery for a long time, the potential difference across the capacitor right after the battery is disconnected is $\Delta V = 15$ V. The electric field strength inside the capacitor is

$$E = \frac{\Delta V_C}{d} = \frac{15\text{ V}}{0.50\times10^{-2}\text{ m}} = 3000\text{ V/m} = 3.0\text{ kV/m}$$

Because $E = \eta/\varepsilon_0$ for a parallel-plate capacitor and $\eta = Q/A$, the total charge on each plate is

$$Q = EA\varepsilon_0 = (3000\text{ V/m})\pi(0.050\text{ m})^2(8.85\times10^{-12}\text{ C}^2/\text{Nm}^2) = 2.1\times10^{-10}\text{ C}$$

(b) After the electrodes are pulled away to a separation of $d' = 1.0$ cm, the charges on the plates are unchanged. That is, $Q' = Q$. Because $A' = A$, the electric field inside the capacitor is also unchanged. So, $E' = E$. The potential difference across the capacitor is $\Delta V_C' = E'd'$. Because d increases from 0.50 cm to 1.0 cm $(d' = 1.0$ cm), the potential difference $\Delta V_C'$ increases from 15 V to 30 V.

(c) When the electrodes are expanded, the new area is $A' = \pi(r')^2 = \pi(2r)^2 = 4A$. The charge Q' on the capacitor plates, however, stays the same as before $(Q' = Q)$. The electric field is

$$E' = \frac{\eta'}{\varepsilon_0} = \frac{Q'}{A'\varepsilon_0} = \frac{Q}{4A\varepsilon_0} = \frac{E}{4} = \frac{3000\text{ V/m}}{4} = 750\text{ V/m} = 0.75\text{ kV/m}$$

The potential difference across the capacitor plates $\Delta V_C' = E'd' = (750\text{ V})(0.050\text{ m}) = 3.8\text{ V}$.

25.61. Model: The electric field inside a capacitor is uniform.

Solve: (a) While the capacitor is attached to the battery, the plates are at the same potentials as the terminals of the battery. Thus, the potential difference across the capacitor is $\Delta V_C = 15$ V. The electric field strength inside the capacitor is

$$E = \frac{\Delta V_C}{d} = \frac{15\text{ V}}{0.50\times10^{-2}\text{ m}} = 3000\text{ V/m} = 3.0\text{ kV/m}$$

Because $E = \eta/\varepsilon_0 = Q/A\varepsilon_0$, the charge on each plate is

$$Q = EA\varepsilon_0 = (3000\text{ V/m})\pi(0.050\text{ m})^2(8.85\times10^{-12}\text{ C}^2/\text{N m}^2) = 2.1\times10^{-10}\text{ C}$$

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(b) After the electrodes are pulled away to a new separation of $d' = 1.0$ cm, the potential difference across the capacitor stays the same as before. That is, $\Delta V_C' = \Delta V_C = 15$ V. The electric field strength inside the capacitor is

$$E' = \frac{\Delta V'}{d'} = \frac{15 \text{ V}}{0.010 \text{ m}} = 1500 \text{ V/m} = 1.5 \text{ kV/m}$$

The charge on each plate is

$$Q' = E'A\varepsilon_0 = (1500 \text{ V})\pi(0.05 \text{ m})^2(8.85\times10^{-12} \text{ C}^2/\text{N m}^2) = 1.0\times10^{-10} \text{ C}$$

(c) The potential difference $\Delta V_C' = \Delta V_C = 15$ V is unchanged when the electrodes are expanded to area $A' = 4A$. The electric field between the plates is

$$E' = E = \frac{\Delta V_C'}{d'} = \frac{15 \text{ V}}{0.50\times10^{-2} \text{ m}} = 3000 \text{ V/m} = 3.0 \text{ kV/m}$$

The new charge is

$$Q' = E'A'\varepsilon_0 = 4EA\varepsilon_0 = 4Q = 8.3\times10^{-12} \text{ C}$$

25.62. Model: The potential energy is determined by the electric potential.
Solve: (a) The electric field in the parallel-plate capacitor when the plates have charges $\pm q$ is

$$E = \frac{\eta}{\varepsilon_0} = \frac{q}{A\varepsilon_0} \Rightarrow \Delta V_C = Ed = \frac{qd}{A\varepsilon_0}$$

The increase in the potential energy dU due to moving charge dq through potential difference ΔV_C is

$$dU = dq\,\Delta V_C = \frac{qd\,dq}{A\varepsilon_0}$$

(b) Integrating the expression for dU from $q = 0$ (uncharged) to $q = Q$ (fully charged),

$$U = \int dU = \int_0^Q \frac{qd}{A\varepsilon_0} dq = \frac{d}{A\varepsilon_0}\int_0^Q qdq = \frac{d}{A\varepsilon_0}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{2}\left(\frac{Qd}{A\varepsilon_0}\right)Q = \frac{1}{2}Q\Delta V_C$$

where we have used the expression for ΔV_C from part (a).

25.63. Solve: (a) Outside a uniformly charged sphere, the electric field is identical to that of a point charge Q at the center. For $r \geq R$,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

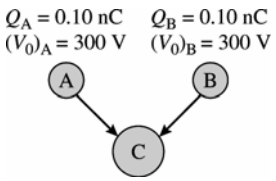
Evaluating this at $r = R$ and using Equation 25.32,

$$E_0 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} = \frac{V_0}{R}$$

(b) The field strength is

$$E_0 = \frac{500 \text{ V}}{0.50\times10^{-2} \text{ m}} = 100{,}000 \text{ V/m} = 100 \text{ kV/m}$$

25.64. Visualize:



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Solve: A sphere of radius R and charge Q has a surface potential $V_0 = Q/4\pi\varepsilon_0 R$. When the two drops A and B merge to form drop C, two quantities are conserved: the total charge and the total volume of mercury. The conservation of charge yields

$$Q_C = Q_{total} = Q_A + Q_B = 0.10 \text{ nC} + 0.10 \text{ nC} = 0.20 \text{ nC}$$

Since the quantity of mercury does not change, the volumes of drops A, B, and C are related by

$$[\text{Vol}_C = \tfrac{4}{3}\pi R_C^3] = [\text{Vol}_A + \text{Vol}_B = \tfrac{4}{3}\pi R_A^3 + \tfrac{4}{3}\pi R_B^2] \Rightarrow R_C = (R_A^3 + R_B^3)^{1/3}$$

We're not given the radii of drops A and B, but we know their surface potentials $(V_0)_A$ and $(V_0)_B$. Thus,

$$(V_0)_A = \frac{Q_A}{4\pi\varepsilon_0 R_A} \Rightarrow R_A = \frac{Q_A}{4\pi\varepsilon_0 (V_0)_A} = \frac{(0.10\times10^{-9} \text{ C})(9.0\times10^9 \text{ N m}^2/\text{C}^2)}{300 \text{ V}} = 3.0\times10^{-3} \text{ m} = R_B$$

Combining these radii gives $R_C = 3.8\times10^{-3}$ m, and

$$(V_0)_C = \frac{Q_C}{4\pi\varepsilon_0 R_C} = \frac{(0.20\times10^{-9} \text{ C})(9.0\times10^9 \text{ N m}^2/\text{C}^2)}{3.8\times10^{-3} \text{ m}} = 0.47 \text{ kV}$$

25.65. Solve: (a) Because a spherical shell of charge has the same electric potential as a point charge Q at the center, the potential on the surface is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \Rightarrow 500{,}000 \text{ V} = (9.0\times10^9 \text{ N m}^2/\text{C}^2)\left(\frac{Q}{0.15 \text{ m}}\right) \Rightarrow Q = 8.3 \ \mu\text{C}$$

(b) The electric field inside a conductor is zero, so the electric field just inside the sphere is zero. From Gauss's Law, the electric field outside a charged conductor is

$$E = \frac{\eta}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{8.33\times10^{-6} \text{ C}}{4\pi(0.15 \text{ m})^2(8.85\times10^{-12} \text{ C}^2/\text{Nm}^2)} = 3.3\times10^6 \text{ V/m}$$

25.66. Model: The potential at any point is the superposition of the potentials due to all charges. Outside a uniformly charged sphere, the electric potential is identical to that of a point charge Q at the center.
Visualize: Sphere A is the sphere on the left and sphere B is the one on the right.
Solve: The potential at point a is the sum of the potentials due to the spheres A and B:

$$\begin{aligned} V_a &= V_{A \text{ at } a} + V_{B \text{ at } a} = \frac{1}{4\pi\varepsilon_0}\frac{Q_A}{R_A} + \frac{1}{4\pi\varepsilon_0}\frac{Q_B}{0.70 \text{ m}} \\ &= (9.0\times10^9 \text{ N m}^2/\text{C}^2)\frac{100\times10^{-9} \text{ C}}{0.30 \text{ m}} + (9.0\times10^9 \text{ N m}^2/\text{C}^2)\frac{25\times10^{-9} \text{ C}}{0.70 \text{ m}} \\ &= 3000 \text{ V} + 321 \text{ V} = 3321 \text{ V} \end{aligned}$$

Similarly, the potential at point b is the sum of the potentials due to the spheres A and B:

$$\begin{aligned} V_b &= V_{B \text{ at } b} + V_{A \text{ at } b} = \frac{1}{4\pi\varepsilon_0}\frac{Q_B}{R_B} + \frac{1}{4\pi\varepsilon_0}\frac{Q_A}{0.95 \text{ m}} \\ &= (9.0\times10^9 \text{ N m}^2/\text{C}^2)\left(\frac{25\times10^{-9} \text{ C}}{0.05 \text{ m}} + \frac{100\times10^{-9} \text{ C}}{0.95 \text{ m}}\right) \\ &= 4500 \text{ V} + 947 \text{ V} = 5447 \text{ V} \end{aligned}$$

Thus, the potential at point b is higher than the potential at a. The difference in potential is $V_b - V_a = 5447 \text{ V} - 3321 \text{ V} = 2126 \text{ V} = 2.1 \text{ kV}$.

Assess: $V_{A \text{ at } a} = 3000$ V and the sphere B has a potential of 225 V at point a. The spherical symmetry dictates that the potential on a sphere's surface be the same everywhere. So, in calculating the potential at point a due to the sphere B we used the center-to-center separation of 1.0 m rather than a separation of 100 cm – 30 cm = 70 cm from the center of sphere B to the point a. The former choice leads to the same potential everywhere on the surface whereas the latter choice will lead to a distribution of potentials depending upon the location of point a. Similar reasoning also applies to the potential at point b.

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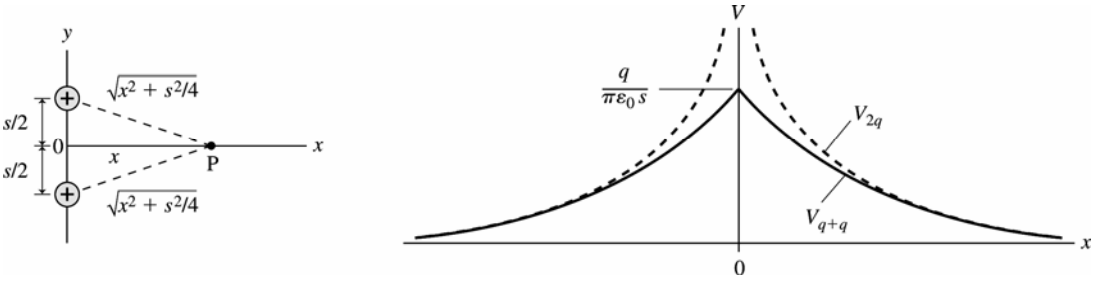
25.67. Model: The potential is the sum of potentials from each charge.
Visualize:



Point P is located on the *x*-axis and the two positive charges are located on the *y*-axis. The separation between the two charges is *s*.
Solve: (a) The potentials at P due to the two positive charges are

$$V_{\text{top}} = \frac{1}{4\pi\varepsilon_0}\frac{(+q)}{\sqrt{x^2 + s^2/4}} \qquad V_{\text{bottom}} = \frac{1}{4\pi\varepsilon_0}\frac{(+q)}{\sqrt{x^2 + s^2/4}}$$

$$\Rightarrow V_{q+q} = V_{\text{top}} + V_{\text{bottom}} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{\sqrt{x^2 + s^2/4}} = \frac{2q}{4\pi\varepsilon_0 x}\frac{1}{\sqrt{1 + s^2/4x^2}}$$

(b) The *V*-versus-*x* graph is shown in the figure. The potential due to a charge $2q$ located at the origin is $V_{2q} = 2q/4\pi\varepsilon_0 x,$ and is also plotted. This expression differs from the expression obtained in part (a) by the factor $(1 + s^2/4x^2)^{-\frac{1}{2}}$. For $x >> s,$ this factor becomes 1 and the two potentials become the same.

25.68. Model: The net potential is the sum of potentials from all the charges.
Visualize: Point P at which we want the net potential due to the *linear electric quadrupole* is far away compared to the separation *s*, that is, $y >> s$.
Solve: The net potential at P is

$$V_{\text{net}} = V_1 + V_2 + V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{y - s} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{y} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{y + s}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{(+q)}{y - s} + \frac{(-2q)}{y} + \frac{(+q)}{y + s}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{y^2 + ys - 2y^2 + 2s^2 + y^2 - ys}{y(y^2 - s^2)}\right] = \frac{q}{4\pi\varepsilon_0}\frac{2s^2}{y(y^2 - s^2)}$$

At distances $y >> s$,

$$V_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\frac{(2qs^2)}{y^3} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{y^3}$$

where $Q = 2qs^2$ is called the electric quadrupole moment of the charge distribution.
Assess: This charge distribution is in fact a combination of two dipoles. As seen above their effects do not completely cancel.

25.69. Model: Assume the thin rod is a line of charge with uniform linear charge density.
Visualize: The point P is a distance *d* from the origin. Divide the charged rod into *N* small segments, each of length Δx and with charge Δq. Segment *i*, located at position x_i, contributes a small amount of potential V_i at point P.
Solve: The contribution of the *i*th segment is

$$V_i = \frac{\Delta q}{4\pi\varepsilon_0 r_i} = \frac{\Delta q}{4\pi\varepsilon_0 (d - x_i)} = \frac{Q\Delta x/L}{4\pi\varepsilon_0 (d - x_i)}$$

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where $\Delta q = \lambda \Delta x$ and the linear charge density is $\lambda = Q/L$. We are placing the point P at a distance d rather than x from the origin to avoid confusion with x_i. The V_i are now summed and the sum is converted to an integral giving

$$V = \frac{Q}{4\pi\varepsilon_0 L}\int_{-L/2}^{L/2}\frac{dx}{d-x} = \frac{Q}{4\pi\varepsilon_0 L}[-\ln(d-x)]_{-L/2}^{L/2} = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{d+L/2}{d-L/2}\right)$$

Replacing d with x, the potential due to a line charge of length L at a distance x along the axis is

$$V = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{x+L/2}{x-L/2}\right)$$

25.70. Model: Assume the thin rod is a line of charge with uniform linear charge density.
Visualize: The point P is located a distance z from the origin. Divide the charged rod into N small segments, each of length Δx and with charge ΔQ. Segment i, located at position x_i, contributes a small amount of potential V_i at point P.
Solve: The contribution of the ith segment is

$$V_i = \frac{\Delta Q}{4\pi\varepsilon_0 r_i} = \frac{\Delta Q}{4\pi\varepsilon_0\sqrt{x_i^2+z^2}}$$

Charge ΔQ is related to length Δx through the linear charge density: $\Delta Q = \lambda\Delta x = (Q/L)\Delta x$. Thus

$$V_i = \frac{Q\Delta x}{4\pi\varepsilon_0 L\sqrt{x_i^2+z^2}}$$

Potential is a scalar, so we can find the net potential at P by summing all the V_i:

$$V = \sum_i V_i = \frac{Q}{4\pi\varepsilon_0 L}\sum_i \frac{\Delta x}{\sqrt{x_i^2+z^2}}$$

If we now let $\Delta x \to dx$, the sum becomes an integral over the length of the rod, from $x_{\min} = -L/2$ to $x_{\max} = +L/2$:

$$V = \frac{Q}{4\pi\varepsilon_0 L}\int_{-L/2}^{L/2}\frac{dx}{\sqrt{x^2+z^2}} = \frac{Q}{4\pi\varepsilon_0 L}\left[\ln\left(\sqrt{x^2+z^2}+x\right)\right]_{-L/2}^{L/2} = \frac{Q}{4\pi\varepsilon_0 L}\ln\left[\frac{\sqrt{z^2+(L/2)^2}+L/2}{\sqrt{z^2+(L/2)^2}-L/2}\right]$$

We can also get a simpler expression by using symmetry to integrate from 0 to $L/2$ and multiplying by 2. This gives

$$V = \frac{Q}{2\pi\varepsilon_0 L}\ln\left[\left(\frac{L}{2z}\right)+\sqrt{1+\left(\frac{L}{2z}\right)^2}\right]$$

This expression can be shown to be equivalent to the one obtained earlier.

25.71. Model: Because the rod is thin, assume the charge lies along the semicircle of radius R.
Visualize: The bent rod lies in the xy-plane with point P as the center of the semicircle. Divide the semicircle into N small segments of length Δs and of charge $\Delta Q = (Q/\pi R)\Delta s$, each of which can be modeled as a point charge. The potential V at P is the sum of the potentials due to each segment of charge.
Solve: The total potential is

$$V = \sum_i V_i = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{R} = \frac{1}{4\pi\varepsilon_0 R}\sum_i\left(\frac{Q}{\pi R}\right)\Delta s = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\pi R^2}\sum_i R\Delta\theta = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\pi R}\sum_i \Delta\theta.$$

All of the terms come to the front of the summation because these quantities did not change as far as the summation is concerned. The summation does not have to convert to an integral because the sum of all the $\Delta\theta$ around the semicircle is π. Hence, the potential at the center of a charged semicircle is

$$V_{\text{center}} = \frac{1}{4\pi\varepsilon_0}\frac{Q\pi}{\pi R} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

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25.72. Model: The disk has a uniform surface charge density $\eta = Q/A = Q/\pi(R_{\text{out}}^2 - R_{\text{in}}^2)$.

Visualize: Orient the disk in the *xy*-plane, with point P at distance *z*. Divide the disk into *rings* of equal width Δr. Ring *i* has radius r_i and charge ΔQ_i.

Solve: Using the result of Example 25.9, we write the potential at distance *z* of ring *i* as

$$V_i = \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q_i}{\sqrt{r_i^2 + z^2}} \Rightarrow V = \sum_i V_i = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{\Delta Q_i}{\sqrt{r_i^2 + z^2}}$$

Noting that $\Delta Q_i = \eta \Delta A_i = \eta 2\pi r_i \Delta r$,

$$V = \frac{1}{4\pi\varepsilon_0}\eta 2\pi \sum_i \frac{r_i \Delta r}{\sqrt{r_i^2 + z^2}} = \frac{\eta}{2\varepsilon_0}\int_{R_{\text{in}}}^{R_{\text{out}}} \frac{r\,dr}{\sqrt{r^2 + z^2}} = \frac{\eta}{2\varepsilon_0}\left[\sqrt{r^2 + z^2}\right]_{R_{\text{in}}}^{R_{\text{out}}} = \frac{\eta}{2\varepsilon_0}\left[\sqrt{R_{\text{out}}^2 + z^2} - \sqrt{R_{\text{in}}^2 + z^2}\right]$$

$$= \frac{Q}{2\pi\varepsilon_0(R_{\text{out}}^2 - R_{\text{in}}^2)}\left[\sqrt{R_{\text{out}}^2 + z^2} - \sqrt{R_{\text{in}}^2 + z^2}\right]$$

In the limit $R_{\text{in}} \to 0$ m,

$$V = \frac{Q}{2\pi\varepsilon_0 R_{\text{out}}^2}\left[\sqrt{R_{\text{out}}^2 + z^2} - z\right]$$

This is the same result obtained for a disk of charge in the example in the chapter.

25.73. Model: Assume the wire is a line of charge with uniform linear charge density. The electric potential at the point is the sum of contributions from all charges present.

Visualize: Let the origin be the center of our coordinate system. Divide the charged wire into two straight sections and the central semicircle. Divide each section into *N* small segments, each of length Δx and with charge Δq. Segment *i*, located at position x_i, contributes a small amount of potential V_i at the semicircle center.

Solve: For the right-hand straight section, the contribution of the *i*th segment is

$$V_i = \frac{\Delta q}{4\pi\varepsilon_0 r_i} = \frac{\Delta q}{4\pi\varepsilon_0 x_i} = \frac{\lambda \Delta x}{4\pi\varepsilon_0 x_i}$$

where $\Delta q = \lambda \Delta x$. The V_i are now summed and the sum is converted to an integral giving

$$V_{\text{straight section}} = \frac{\lambda}{4\pi\varepsilon_0}\int_R^{3R}\frac{dx}{x} = \frac{\lambda}{4\pi\varepsilon_0 R}[\ln(x)]_R^{3R} = \frac{\lambda}{4\pi\varepsilon_0 R}\ln(3)$$

The integration limits are set by the physical location of the straight section. By symmetry, the left-hand straight section adds the same amount to the total potential at the semicircle center.

The potential due to the *i*th segment of the semicircular section is

$$V_i = \frac{\Delta q}{4\pi\varepsilon_0 r_i} = \frac{\Delta q}{4\pi\varepsilon_0 R} = \frac{\lambda(R\,\Delta\theta)}{4\pi\varepsilon_0 R} = \frac{\lambda}{4\pi\varepsilon_0 R}\Delta\theta$$

where we have used the arc length $\Delta x = R\,\Delta\theta$. The V_i are now summed and the sum converted to an integral giving

$$V_{\text{semicircle}} = \frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi} d\theta = \frac{\lambda\pi}{4\pi\varepsilon_0 R} = \frac{\lambda}{4\varepsilon_0 R}$$

The total potential is the sum of the potentials due to the three sections:

$$V = V_{\text{semicircle}} + 2V_{\text{straight section}} = \frac{\lambda}{4\varepsilon_0 R} + 2\left(\frac{\lambda}{4\pi\varepsilon_0 R}\ln(3)\right) = \frac{\lambda}{4\varepsilon_0 R}\left(1 + \frac{2\ln(3)}{\pi}\right)$$

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25.74. Solve: (a) A charge of 40 nC is separated into two charges that are placed 3.0 cm apart. The potential energy of the charge arrangement is 90 μJ. What are the two charges?

(b) We have $q_1q_2 = 3.0\times10^{-16}\ \text{C}^2$ and $q_1 + q_2 = 40\times10^{-9}\ \text{C}$. Substituting into the first equation an expression for q_1 from the second equation,

$$(40\times10^{-9}\ \text{C} - q_2)q_2 = 3.0\times10^{-16}\ \text{C}^2 \Rightarrow q_2^2 - (40\times10^{-9}\ \text{C})q_2 + 3.0\times10^{-16}\ \text{C}^2 = 0$$

The solutions to this equation are $q_2 = 30$ nC and 10 nC. This means the two charges are 10 nC and 30 nC.

25.75. Solve: (a) A proton is shot toward a 2.0 nC point charge with a speed of 2.5×10^6 m/s. What is the proton's speed when it is 1.0 mm from the charge?

(b) Solving the equation yields $v_i = 1.67\times10^6$ m/s.

25.76. Solve: (a) Two 3.0 nC charges are distance *d* apart. At a point 3.0 cm from one charge, on the side opposite the other charge, the potential is 1200 V. Find the separation *d* between the charges.

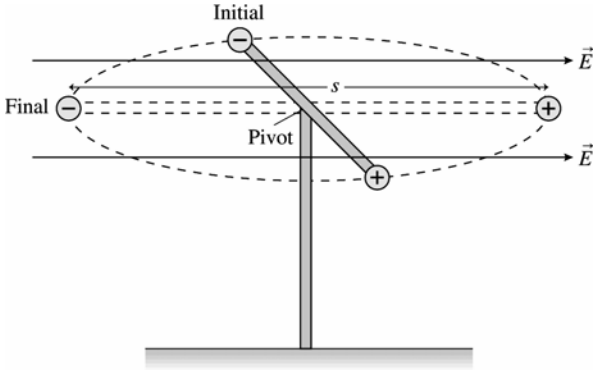
(b) The given equation simplifies to

$$900\ \text{N m/C} + \frac{27\ \text{N m}^2/\text{C}}{0.030\ \text{m} + d} = 1200\ \text{V} \Rightarrow \frac{27\ \text{N m}^2/\text{C}}{0.030\ \text{m} + d} = 300\ \text{V} \Rightarrow d = 0.060\ \text{m} = 6.0\ \text{cm}$$

Challenge Problems

25.77. Model: Energy is conserved.

Visualize:



The dipole initially is at the higher potential energy. As it moves to its minimum energy state, the decrease in the potential energy appears as rotational kinetic energy.

Solve: The potential energy of a dipole is $U_{\text{dipole}} = -\vec{p}\cdot\vec{E} = -pE\cos\theta$, where θ is the angle between the dipole and the electric field. The energy conservation equation $K_\text{f} + U_\text{f} = K_\text{i} + U_\text{i}$ is

$$\tfrac{1}{2}I\omega^2 + U_\text{f} = 0\ \text{J} + U_\text{i} \Rightarrow \tfrac{1}{2}I\omega^2 = U_\text{i} - U_\text{f} = -pE\cos90° - (-pE\cos0°)$$

$$\Rightarrow \tfrac{1}{2}I\omega^2 = pE \Rightarrow \omega = \sqrt{\frac{2pE}{I}}$$

The moment of inertia of the dipole is $I_{\text{dipole}} = 2m\left(\tfrac{1}{2}s\right)^2$. Substituting into the above equation,

$$\omega = \sqrt{\frac{2(qs)E}{2m\left(\frac{1}{2}s\right)^2}} = \sqrt{\frac{4(2.0\times10^{-9}\ \text{C})(0.10\ \text{m})(1000\ \text{V/m})}{(1.0\times10^{-3}\ \text{kg})(0.10\ \text{m})^2}} = 0.28\ \text{rad/s}$$

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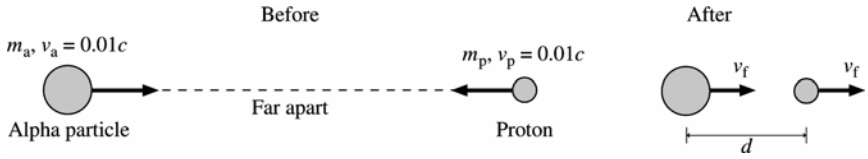
25.78. Model: Apply the principles of conservation of energy and conservation of momentum.
Visualize:



The figure shows a before-and-after pictorial representation of the charged particles moving toward each other. The proton's physical quantities are denoted by the subscript p and that of the alpha particle by the subscript a.
Solve: The conservation of energy equation is $K_f + U_f = K_i + U_i$. Initially when the charges are far away, $U_i = 0$ J and $K_i = \frac{1}{2}m_p v_{ip}^2 + \frac{1}{2}m_a v_{ia}^2$. At the distance of closest approach,

$$U_f = \frac{1}{4\pi\varepsilon_0}\frac{(e)(2e)}{d} \qquad K_f = \tfrac{1}{2}m_a v_{fa}^2 + \tfrac{1}{2}m_p v_{fp}^2$$

The particles are *not* at rest at closest approach. That would violate conservation of momentum. Instead, the particles are instantaneously moving with the same velocity $v_{fa} = v_{fp} = v_f$ so that the distance between them is (instantaneously) not changing. The conservation of energy equation simplifies to

$$\tfrac{1}{2}(m_a + m_p)v_f^2 + \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{d} = \tfrac{1}{2}(m_p + m_a)(0.010c)^2$$

The conservation of momentum equation $P_{after} = P_{before}$ is

$$m_a v_f + m_p v_f = m_a(0.010c) - m_p(0.01c)$$

$$\Rightarrow v_f = \frac{(0.010c)(m_a - m_p)}{m_a + m_p} = \frac{(0.010)(3\times10^8\ \text{m/s})(4\ \text{u} - 1\ \text{u})}{4\ \text{u} + 1\ \text{u}} = 1.80\times10^6\ \text{m/s}$$

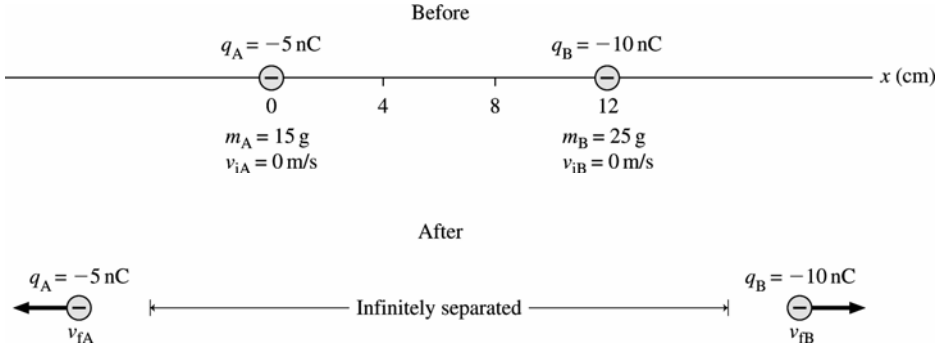
Substituting into the energy-conservation equation,

$$\tfrac{1}{2}(4\ \text{u} + 1\ \text{u})(1.80\times10^6\ \text{m/s})^2 + (9.0\times10^9\ \text{N m}^2/\text{C}^2)\frac{2(1.60\times10^{-19}\ \text{C})^2}{d} = \tfrac{1}{2}(4\ \text{u} + 1\ \text{u})(3.0\times10^6\ \text{m/s})^2$$

$$\Rightarrow \frac{4.608\times10^{-28}\ \text{N m}^2}{d} = 1.44\times10^{13}\ \text{u m}^2/\text{s}^2$$

$$\Rightarrow d = \frac{4.608\times10^{-28}\ \text{N m}^2}{(1.44)(1.661\times10^{-27}\ \text{kg})\times10^{13}(\text{m}^2/\text{s}^2)} = 1.9\times10^{-14}\ \text{m}$$

25.79. Model: The charged beads are point charges.
Visualize:



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Solve: To find the maximum speeds v_{fA} and v_{fB} it is easier to use the conservation of momentum and conservation of energy equations than kinematics, as suggested in the hint. The beads achieve their maximum speeds when they are infinitely separated. The momentum conservation equation along *x*-direction $p_{after} = p_{before} = 0$ kg m/s means

$$-m_A v_{fA} + m_B v_{fB} = 0 \text{ kg m/s} \Rightarrow -(0.015 \text{ kg})v_{fA} + (0.025 \text{ kg})v_{fB} = 0 \text{ kg m/s} \ = 0 \text{ kg m/s} \Rightarrow v_{fB} = \tfrac{3}{5}v_{fA}$$

The conservation of energy conservation is $U_f + K_f = U_i + K_i$. Noting that $U_f \to 0$ J as the masses are infinitely separated and

$$U_i = \frac{1}{4\pi\varepsilon_0}\frac{Q_A Q_B}{r_{AB}} = \frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(5.0\times10^{-9} \text{ C})(10.0\times10^{-9} \text{ C})}{0.12 \text{ m}} = 3.75\times10^{-6} \text{ J}$$

$$\Rightarrow 0 \text{ J} + (\tfrac{1}{2}m_A v_{fA}^2 + \tfrac{1}{2}m_B v_{fB}^2) = 3.75\times10^{-6} \text{ J} + 0 \text{ J}$$

$$\Rightarrow \tfrac{1}{2}(0.015 \text{ kg})v_{fA}^2 + \tfrac{1}{2}(0.025 \text{ kg})\left(\tfrac{3}{5}v_{fA}\right)^2 = 3.75\times10^{-6} \text{ J} \Rightarrow 0.012 v_{fA}^2 = 3.75\times10^{-6} \text{ J}$$

Solving these equations yields $v_{fA} = 0.0177$ m/s ≈ 0.018 ms and $v_{fB} = \frac{3}{5}(0.0177 \text{ m/s}) = 0.011$ m/s. These are the maximum speeds of the two beads and occur when they are infinitely separated from each other.

25.80. Model: Energy and momentum are conserved.
Visualize: Let the two spheres have masses m_C and m_D, and speeds v_C and v_D when the two spheres collide.
Solve: The energy-conservation equation $K_f + U_f = K_i + U_i$ is

$$\left(\tfrac{1}{2}m_C v_C^2 + \tfrac{1}{2}m_D v_D^2\right) + \left(\frac{1}{4\pi\varepsilon_0}\frac{q_C q_D}{0.002 \text{ m}}\right) = 0 \text{ J} + \frac{1}{4\pi\varepsilon_0}\frac{q_C q_D}{0.010 \text{ m}}$$

The momentum conservation equation $P_{after} = P_{before}$ is

$$m_C v_C + m_D v_D = 0 \text{ kg m/s} \Rightarrow v_C = -\left(\frac{2.0 \text{ g}}{1.0 \text{ g}}\right)v_D = -2v_D$$

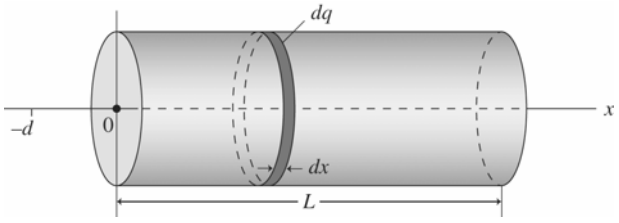
Substituting this expression in the energy equation,

$$\tfrac{1}{2}(0.001 \text{ kg})4v_D^2 + \tfrac{1}{2}(0.002 \text{ kg})v_D^2 = (9.0\times10^9 \text{ N m}^2/\text{C}^2)(2.0\times10^{-9} \text{ C})(-1.0\times10^{-9} \text{ C})\left[\frac{1}{0.010 \text{ m}} - \frac{1}{0.002 \text{ m}}\right]$$

$$\Rightarrow (0.0030 \text{ kg})v_D^2 = 7.2\times10^{-6} \text{ N m}$$

Solving these equations gives $v_D = 0.049$ m/s $= 4.9$ cm/s and hence $v_C = -9.8$ cm/s. The speed of C is +9.8 cm/s.

25.81. Model: The total charge *Q* is the sum (integral) of the small (differential) elements *dq*.
Visualize: We are given $\lambda(x) = \lambda_0 x/L$.



Solve: (a) Integrate from 0 to *L*.

$$Q = \int dq = \int \lambda dx = \int_0^L \lambda_0 \frac{x}{L}dx = \frac{\lambda_0}{L}\int_0^L x dx = \frac{\lambda_0}{L}\left[\tfrac{1}{2}x^2\right]_0^L = \tfrac{1}{2}\lambda_0 L \Rightarrow \lambda_0 = \frac{2Q}{L}$$

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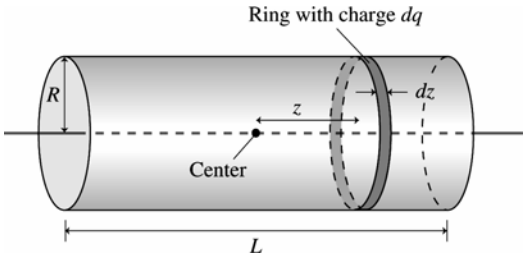
(b) Each tiny slice of the rod has a charge dq and contributes dV to the potential at the point d to the left of the rod. So we integrate from 0 to L where the distance from the point to dq is $r = x + d$.

$$V = \int dV = \int K\frac{dq}{r} = \int K\lambda\frac{dq}{r} = K\int_0^L\left(\lambda_0\frac{x}{L}\right)\frac{dx}{x+d} = \frac{K\lambda_0}{L}\int_0^L\frac{x}{x+d}dx = \frac{K\lambda_0}{L}\Big[x - d\ln(x+d)\Big]_0^L$$
$$= \frac{K\lambda_0}{L}\Big[L - d\ln\big(L+d\big) + d\ln d\Big] = \frac{K\lambda_0}{L}\left[L + d\ln\left(\frac{d}{L+d}\right)\right]$$

Assess: The potential depends directly on λ_0, as we expect. We also see that the answer behaves as expected as $d \to 0$ and as $d \to \infty$.

$$V = \frac{3Q}{8\pi\varepsilon_0 R^3}\left(R\sqrt{R^2+z^2} + \ln\left(\frac{|z|}{R+\sqrt{R^2+z^2}}\right)\right)$$

25.82. Model: Assume the cylinder is uniformly charged.
Visualize: Divide the cylinder into narrow rings of width dz, radius R, and charge dq.



Solve: The potential at the center is the sum of the potential of all the rings. The potential at distance z on the axis of a ring of charge was found in Example 25.9 to be

$$V_{\text{ring}} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{R^2+z^2}}$$

The surface area of the ring is $dA = (2\pi R)dz$, so the amount of charge is $dq = \eta\, dA = (2\pi R\eta)dz$. The surface area of the cylinder is $A = (2\pi R)L$, so the surface charge density is $\eta = Q/2\pi RL$. Thus $dq = (Q/L)dz$ and the potential of the ring is

$$V_{\text{ring}} = \frac{Q/L}{4\pi\varepsilon_0}\frac{dz}{\sqrt{R^2+z^2}}$$

The potential at the center of the cylinder is found by summing (i.e., integrating) the potential due to all rings. That is,

$$V_{\text{cyl}} = \frac{Q/L}{4\pi\varepsilon_0}\int_{-L/2}^{L/2}\frac{dz}{\sqrt{R^2+z^2}} = \frac{2Q/L}{4\pi\varepsilon_0}\int_0^{L/2}\frac{dz}{\sqrt{R^2+z^2}} = \frac{2Q/L}{4\pi\varepsilon_0}\left[\ln\left(z+\sqrt{R^2+z^2}\right)\right]_0^{L/2}$$
$$= \frac{2Q/L}{4\pi\varepsilon_0}\ln\left(\frac{L}{2R} + \sqrt{1+\left(\frac{L}{2R}\right)^2}\right)$$

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